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AUTHOR Crawford, Douglas H.
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ABSTRACT

This study was designed to: (1) outline briefly the meaning and importance of the family of concepts stemming from the ideas of estimation, approximation, checking, and precision; (2) survey the present place of such concepts in the curriculum; and (3) relate the teaching and learning of these concepts to the availability of the calculator as an emerging tool for reducing computational labor. Relevant literature was extensively surveyed: the meanings of the terms were analyzed; their role in the curriculum was traced; and the use of the hand-held calculator was addressed. The following section discusses the literature under the headings of meaning, rationale, research, the curriculum, and teacher strategies. The present place of these concepts in the Ontario school mathematics curriculum was analyzed using surveys of mathematics coordinators and comparisons of textbooks and other documents. Results of the questionnaire survey and document analyses are found in the next section, followed by a brief summary. A bibliography is included, as well as documents related to the survey. (MNS)

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A Pilot Study of the Significance of Estimation, Approximation and Related Concepts in the School Mathematics Curriculum

by Douglas H. Crawford

Report of Research
Funded by the Faculty of Education
Queen's University at Kingston

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I. ORIGIN, AIMS AND PLAN OF THE STUDY

I.1 Origin and Aims

The study arose as a natural outgrowth of a previous study on Basic Skills in Mathematics (Crawford 1980 a; b; c) which involved a study of the literature on this theme in England, Canada and the U.S.A. Thus the Working Group on goals in the 1975 Euclid Conference on Basic Mathematical Skills identified Estimation and Approximation as one of ten Basic Goals.

> "These skills (estimation and approximation) are basic to facility and comfort with quantitative ideas. Students should know some simple techniques for estimating quantity, length, distance, weight, and so on. Also, students should be able to carry out approximate, rapid calculations by first rounding off numbers. Necessary here are a sense of the likely error in various procedures, and of whether a particular result is precise enough for the purpose at hand."

They also included Alertness to Reasonableness of Results as another of their ten goals.

> "Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Occasionally they are manifestly unsound. Students should learn to inspect all results, checking for reasonableness in terms of the original problem."

In almost the same words, the National Council of Supervisors of Mathematics included both of these ares in its famous position

paper on Ten Basic Skill Areas in October 1977, while the National council of Teachers of Mathematics An Agenda for Action - Recommendations for School Mathematics of the 1980's endorses the NCSM statement and recommends changes in the priorities and emphases in the instructional program in order to reflect the expanded concept of basic skills implied by the scope of the skills so identified.

The importance of the calculator as an emerging ubiquitous tool whose place in the curriculum requires sustained research and experimentation was also acknowledged in the N.I.E. Goals statement at the Euclid Conference, the NCSM position statement and the NCTM 1980 An Agenda for Action.

In the NCTM 1980 statement, calculators and computers get a full recommendation all to themselves with eleven recommended actions including the need to develop and make available "curriculum materials that integrate and require the use of the calculator and computer in diverse and imaginative ways".

Additionally, the Ontario Guidelines for the Intermediate Mathematics Curriculum released in 1980 accepts the calculator as a valuable aid in applying mathematics to the environment (page 3) and indicates how calculators can not only remove much of the drudgery of mathematics but can help to "introduce, reinforce or illuminate new concepts". It specifically mentions checking oral and written exercises, applying estimation and rounding skills, and investigating approximation errors resulting from estimation and rounding (p.11).

The study was conceived therefore with the following aims:

(1) to outline briefly the meaning and importance of the family of concepts stemming from the ideas of estimation, approximation, checking and precision.

(2) to survey the present place of such concepts in the curriculum by a study of present Ministry of Ontario guidelines, curricula currently used by selected Ontario Boards, and relevant mathematics texts and other documents.

(3) to relate the teaching and learning of these concepts to the availability of the hand-held calculator as an emerging tool for reducing computational labour and assisting in the better understanding of them.

I.2 The Plan of the Study

The study proceeded in two parallel ways. Firstly, a survey was made of the relevant literature. Secondly, a study was made of the present place of these concepts in the Ontario school mathematics curriculum by:

(i) developing and analysing a 4-part questionnaire given to members of the Ontario Mathematics Coordinators Association

(ii) collecting and analysing relevant Ministry of Ontario and local board of Education documents and guidelines, together with selected texts used by a significant number of the Boards.

Additionally, by consulting dictionaires, and references in the literature, the actual meaning of the two concepts estimation and approximation was studied in order to ensure that the basis on which conclusions and/or recommendations were made would be sound.

In practice, the literature study was done in two parts - one in early 1981, the other, more systematically in early summer 1982. The questionnaire part of the study proceeded well on schedule until summer 1981, but final analysis and interpretations then had to wait until summer 1982.

II LITERATURE SURVEY

Two ordinary dictionaries and one mathematical one were consulted as just indicated, so that the use of the terms 'estimation' and 'approximation' as they appeared in the literature could be analysed. Additionally, a wide search of the relevant literature was conducted. The sources which proved most valuable were N.C.T.M. Yearbooks, Conference Reports, Journal Articles and Books. These included a number fo research studies.

II.1 Dictionary Meanings of the terms 'Estimation' and 'Approximation"

Two normal dictionaries and one mathematical dictionary were consulted to gain some basic information on the meaning normally assigned to these terms.

According to Funk and Wagnall (1956), the following meanings

and definitions were assigned:

ESTIMATION 1. Estimate (verb) -

(i) to calculate; compute

(ii) to assign a value to; rate

(iii) to form a general opinion about

2. Estimate (noun) - A valuation based on opinion or roughly made from imperfect or incomplete data; a calculation not professedly exact; appraisement. Carefully weighed judgment.

3. Estimation (noun) -

(i) the act of estimating

(ii) the conclusion arrived at

Derivation: aestimo - value

APPROXIMATION

1. Approximate (verb) - to approach or cause to approach closely, without exact coincidence.

2. Approximate (adjective) - nearly, but not exactly accurate or complete

3. Approximation (noun) -

(i) the act or process of approximating

(ii) an approximate result

Derivation: proximus - nearest

Webster (1965) gave the following meanings

ESTIMATION 1. Estimate (verb) -

(i) to judge tentatively or approximately the

value, worth, or significance of

(ii) to determine roughly the size, extent, or nature of

(iii) to produce a statement of the approximate cost of

2. Estimate (noun) -

(i) the act of appraising or valuing

(ii) an opinion or judgement of the nature, character or quality of a thing

(iii) a rough or approximate calculation

3. Estimation (noun) -

(i) the act of estimating

(ii) estimate

Derivation: aestimare - to value, estimate

APPROXIMATION

1. Approximate (adjective) -

(i) nearly correct or exact

(ii) located close together

2. Approximate (verb) -

(i) to bring near or close

(ii) to come near to

3. Approximation (noun) -

(i) the act or process of drawing together

(ii) the quality or state of being close, especially in value

(iii) something that is approximate, especially a

mathematical value that is nearly but not exactly correct

Derivation: proximare - to come near

In Van Nostrand's Scientific Encyclopaedia, Third Edition (1958), there is no entry for "estimation", but for "approximate calculation" the entry is as follows:

APPROXIMATE CALCULATION -

It is often necessary, especially in physical problems, to make approximate, rather than exact, calculations. Typical cases include: the solution of polynomial or transcendental equations; evaluation of integrals which cannot be given in terms of simple functions; solution of differential equations with assigned boundary conditions.

Sometimes graphical or mechanical methods are convenient and these, while limited in their precision, frequently yield results of complete satisfaction. Numerical methods are more generally applicable, for with desk calculating machines an answer can usually be obtained with any reasonable number of significant figures desired. In these cases, the procedure can be classified as one of two general classes: polynomial approximation or iteration.

The literature of this subject is very large and

only a brief survey of it can be given here. Moreover, the methods involved can be modified in many different ways and there is no generally accepted name given to the various procedures. If an approximate calculation is to be made, however, it is hoped that some help may be found in one or more of the following references: curve fitting; differential equation, numerical solution of; differentiation, numerical; equation, equation, numerical solution of; graphical methods; integration, numerical; interpolation.

II.2 N.C.T.M. Yearbooks

1. Approximate Computation (12th - 1937)

As the writer, Aaron Bakst, states in the Preface, the aim of this work is

"to present a rigorous treatment of the methods of computation with approximate numbers"

The work is in two parts; Part I discusses in detail the teaching of approximate computation while Part II is devoted to the development of the Theory of Numerical Processes which underlies the theory of Approximate Computation.

Bakst first summarises the 1923 Report of the National Committee on Mathematical Requirements which recommended the inclusion of topics on approximation in secondary school mathematics and made general suggestions as to how this might be done. However, an analysis by him (Bakst) of 88 Algebra and 34

Trigonometry textbooks indicated a surprising lack of work on the theme, and numerous other weaknesses and inadequacies, e.g., in only eight algebra texts is the idea of measurement introduced, and only one of these mentions the idea of an error in measurement. Little was being done during this period (1923-32) in other countries examined (Soviet Russia, Germany, France, Great Britain).

The author stresses the need for both textbook writer and teachers to be thoroughly versed in the subject. He claims that

> "in teaching the representation of approximate numbers one should emphasize the idea of a measurement. The numerical representation of a measure can be connected with the notion of measurement only by stating an approximate number in one of these three ways: (1) as correct to a certain unit (or units) of measurement, that is, hundreds, tens, hundredths, thousandths, and so on; (2) as correct within a stated error of measurement; (3) as correct to a certain number of significant digits. The so-called decimal place approximativeness is of very little value in Approximate Computation either from the point of view of conveying direct information or from the point of view of the practical numerical processes, and it should never be used." (page 15)

He then classifies Approximate Computation into 2 types as follows:

Type 1: The nature of the numerical data is definitely

given, and it is required to apply the formulas and to perform the numerical computations in such a manner that the optimum approximation possible is obtained after all the stated computations are performed.

Type 2: The nature of the numerical data is definitely given, and it is required to apply the formulas and to perform the numerical computations in such a manner that a result with a definitely predetermined approximativeness is obtained after all the stated computations are performed.

He then proceeds to outline how approximate computation should be taught by dividing it into the following seven major topics:

I. Measurement.
II. Approximate numbers.
III. Numerical operations.
IV. Logarithms.
V. Natural trigonometric functions.
VI. Approximate Computation of the first type.
VII. Approximate Computation of the second type.

He next devotes a chapter to each of these topics with a suggested development supplemented by "typical illustrative material and extensive teaching remarks".

The basis for the teaching outlined in Part I is developed in Part II.

For example, he points out that errors in and of measurements have what he calls the accumulation property, and should always

be added. He deals very clearly with the meaning and nature of exact and approximate qualities, and with the concept of approximativeness (where $\bar{y}$ is an approximation of $\bar{y}$ with a certain degree of approximativeness. The excess or deficit of the approximation y is the error).

This is a very complete and scholarly work, which is still a prime reference for anyone who wishes to be clear on the foundations whereby meaningful approximations are obtained from all numerical calculations involving numerical data. Particularly interesting, for example, are the discussions of error in measurement, significant digits and rounding off digits in Chapter V, with the corresponding reference to theory on pp. 112-125, and Appendix I).

2. Emerging Practices in Mathematics Education (22nd-1954)

Two excellent articles are included. In the first Carl Shuster begins by emphasizing that we live in a world of approximate data, that measurement itself is never exact, and that many sources of approximate data exist. He goes on to discuss the basis, and procedures for teaching computation using approximate data. He relates the discussion to measuring, and defines unit of measurement, precision of measurement, significant digit and rounding, so that he can apply these to the actual process of computing with approximate data. He also distinguishes clearly between precision and accuracy as follows.

Precision of a measurement is determined by the unit of

be added. He deals very clearly with the meaning and nature of exact and approximate qualities, and with the concept of approximativeness (where $\bar{y}$ is an approximation of $\bar{y}$ with a certain degree of approximativeness. The excess or deficit of the approximation y is the error).

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Precision of a measurement is determined by the unit of

measurement. It is greater the smaller the unit, whereas accuracy is determined by the number of significant digits. Digits in a measurement are significant in that the last of them should be at the same "level" as the unit of measurement. Consider the measurements 0.0064 and 2.43126. These are equally precise because the same unit of measurement was used, but the second is far more accurate because it has 6 as opposed to 2 significant digits. To put it another way, the relative error in the measurement is much smaller.

The use of zeros is also vitally important. Zeros which are the result of correct measurement are significant, whereas zeros used to give the correct place value in a rounded number are not. (300 m 300 0.5 m, whereas 186,000 rounded from 186,284 is 186,000 0.5)

Finally, Shuster emphasizes the difference between exact and approximate numbers, and how to deal with them in computations.

Gagner, in the second article, takes up where Shuster leaves off, and defines exact numbers as those obtained by counting units whereas approximate numbers are those obtained by measurement or rounding. He goes on to discuss error, precision, significant digits and accuracy, thus enabling him to define in turn relative and per cent error, and degree of accuracy. He concludes by discussing computation, including computation of angular measurements, and makes some suggestion for grade placement beginning with distinguishing between exact and approximate numbers in Grade 7.

3. The Growth of Mathematical Ideas (24th-1959)

Chapter 5 of this Yearbook, by Payne and Seber, looks at Measurement and Approximation very systematically and comprehensively, dealing in turn with estimation, measurement, and approximation. Each section traces the development and use of the specific concept discussed from an early age to the end of high school.

With respect to estimating, examples are given of its value in computation throughout the grades, including the estimation of as about 3, and the use of estimaton when finding square roots. Behind this discussion lies the importance of developing the concepts of order relations (more than, less than, and between). There is also useful discussion of estimating the number of objects, e.g., about how many people are there at the meeting, and estimating linear and other similar measurements.

In the section on measurement, various useful relevant ideas are considered. These include direct and indirect measurement, the variety of units (very small to very large), the relationship between units, the tolerance (greatest possible error) of a measurement, and the need to acquaint pupils with standard ways in which precision of measurement is indicated. (e.g., and standard notation). It is also pointed out that in fact, no number is itself approximate, it is the source or use of the number (measurement, or need to approximate) which makes it so. Perhaps we should use the phrase approximate data to emphasize

this.

A third section deals with computation with numbers from approximations. Excellent examples are given to show that accuracy is independent of the unit of measurement, and there are clear discussions of significant digits and precision, and the fact that accuracy of a measurement cannot be improved by computation.

Finally, there is a fascinating discussion of the concept of approximation, beginning from the basic assumption that "the continued study of mathematics demands that more and more attention be given to a distinction between mathematical conditions and the physical situations which they describe." In turn, various facets of the concept are explored. The use of division to begin the study of rational approximation and the understanding that approximation is a relation between two numbers is excellently developed in both numerical and geometric contexts. Consideration of the length of the diagonal of a square can lead to the idea of successive approximation, and hence to limits and calculus. A more abstract approach to order on a line can yield deep understandings of the idea of closeness, and in particular, the realisation that things close to the same thing may not be close to each other. The analogous concepts of 'is close to' and 'is approximately' can be developed further using graphing in the Cartesian plane, with important later application in such areas as linear programming, and science generally.

Three common scientific situations in which approximation is involved are (i) the use of mathematical conditions which involve numbers for which there is no terminating decimal representation e.g., 3 or 5/7, (ii) the use of mathematical or physical tables and (iii) the use of drawings, graphs, and data to suggest mathematical conditions which define relations and functions. Numerous examples are given which illustrate these three situations, especially the application to regions in the Cartesian plane defined by inequalities. For example $y-1 < mx-b < y+1$ as an approximation to points satisfying $y = mx + b$ can serve a prototype for meaningful use and practice with quadratic and other functions.

4. Measurement in School Mathematics (1976)

Chapter 5 entitled "Estimation as Part of Learning to Measure" by Bright focusses on estimating measurements made up of both a number and a unit, since this is the most common use of estimating. Three areas are examined - the relationship between estimating and measurement; illustrations of ways of teaching estimation; and sequencing of estimating activities in instruction at all levels.

Estimating a measurement is defined as the process of arriving at a measurement without the aid of measuring tools, and so is seen as a mental process with several accompanying ideas (p. 89). Eight different kinds of estimation are identified, four in each of two classes A, and B. In A, both the property (e.g.,

length) and the object to be measured are named, e.g., estimate the volume of a garbage can in liters. In B, the measure is given, and the object which has about that measure is to be named or identified, e.g., Which of the following would normally have a temperature closest to 5°C? Ice cube, candle flame, Gulf of Mexico.

Giving exercises with objects and units absent as well as present helps to develop mental pictures of units of measure, and sharpens the student's ability to estimate using mental images.

Teaching estimation involves knowing why it is useful, and also developing meaningful activities at appropriate ages and stages in the curriculum. The major aim should be to enable students to develop skills which they can apply in a variety of situations - first in developing and consolidating ideas of number and numerousness, then in computation, and more generally in real life situations to understand, describe, and control recurrent problem situations. Care should be taken to evaluate skills as they are developed and taught.

The general rule is estimate first, then measure. Frames of reference need to be established for each major unit of measure; (length, volume, mass, temperature) then activities involving the right kinds of estimation should be provided in a systematic planned sequence. Games also provide an excellent medium for development and practice, and the focus should be on situations similar to those encountered in the real world.

5. Developing Computational Skills (1978)

In Chapter 12 of this yearbook, Trafton discusses Estimation and Mental Arithmetic as Important Components of Computation. Many situations arise in everyday life where the ability to use estimation skills mentally is extremely helpful. Such skills should form part of the minimal competency in mathematics expected of all citizens.

Four strategies for developing estimating skills are presented (pp. 200-207). Included are the use of rounded numbers, a reference point, front-end estimation (going from largest to smallest place values), and estimating ranges of an answer, which requires understanding of significant digits (p. 205). This last type of estimating assures greater importance now that hand-held calculators are in common everyday use.

The most useful mental arithmetic skills, according to Trafton are applications of front-end estimation to addition, subtraction, and multiplication. To assist in developing these skills, he makes suggestions for using physical and visual models (pp. 209, 210). Finally, he proposes a set of guidelines, which emphasize teaching meaningfully, using applied situations and stressing oral work.

6. Developing Mental Arithmetic (Atweh, 1982 Yearbook)

In a contribution to the 1982 N.C.T.M. Yearbook which has the theme Mathematics for the Middle Grades (5-9), Atweh stresses "The importance of developing the appropriate skills to

enhance mental arithmetic, including some aspects of estimation, and the relationship of these skills to the middle grade mathematics curriculum."

He first illustrates how mental methods differ from the normal computational algorithms, and contrasts the two concepts - mental arithmetic and estimation. He considers these two independent dimensions of computation - one referring to method, the other to "precision as opposed to an exact answer or solution". (p. 51)

He then examines the role of mental arithmetic in the school curriculum, and the various reasons and purposes advanced for including it. He justifies its importance indicating three real- life aspects which depend on it rather than one exact computation. These are:

(i) many real life situations exist where computational tools (calculator, pencil/paper) are not available.

(ii) many real life problems do not need exact answers

(iii) the magnitude of error in some real-life computations could be rather crucial. (Mental Arithmetic can provide a check on either accidental or conceptual errors).

Finally, he considers implications for the curriculum, suggesting that in particular there needs to be emphasis on understanding, problem-solving, reflection and sequencing and concludes by listing some sample mental arithmetic activities.

This is a refreshing article, containing some excellent thinking, and a useful follow-up bibliography.

II.3 CONFERENCE REPORTS AND JOURNAL ARTICLES

1. Four Relevant Research Studies

(a) The Effect of the Teaching of Estimation on Arithmetic Achievement in the Fourth and Sixth Grade (Nelson, 1966)

This research study tested whether the teaching of estimation would improve arithmetic competencies and estimation competencies on the basis of one week's teaching of estimation to an experimental group which was compared with a control group - at both Grade 4 and 6 levels. No significant differences in arithmetic competencies were found but

> Children given instruction and practice in the application of estimation became more competent with this procedure than those who had not received such instruction. Those children in the experimental group who had the highest intelligence quotients generally made the highest scores on the test of estimation. Ability in estimation seemed to be more related to the ability to apply arithmetic procedures in social situations than to ability to compute. The results showed that it was difficult to integrate a skill such as estimation into a program in one year. Therefore, it seems unlikely that children will noticeably change arithmetic behavior in such a short time due to previous habits and teaching.

(b) The Ability to Estimate in Mathematics (Paull, 1971)

In this study, an attempt was made to analyze the ability to

estimate length, area and the answers to numerical computations using students in twelve college preparatory eleventh grade mathematics classes. The procedure was to randomize 196 students into three testing groups each group taking two of three test batteries.

The conclusions included the following:

1. Subjects are not consistent across tasks in their ability to estimate answers to problems.

2. The ability to estimate answers to arithmetic problmes is positively correlated with mathematical ability and verbal ability.

3. The ability to estimate is modestly, but significantly, correlated with mathematical ability and verbal ability.

4. The ability to estimate length is not correlated with either mathematical ability or verbal ability.

5. The ability to estimate area and the ability to estimate length are not correlated with the ability to do mathematical computations rapidly.

6. There is a positive correlation between the ability to estimate answers to numerical computations and the ability to solve mathematics problems by trial and error.

7. The ability to estimate numerical computations is a significant predictor of the ability to solve problems by trial and error.

(c) <u>The Effect of Using Estimation in Learning a Unit of Sixth Gra</u>

Mathematics (Ibe, 1971)

The effect of using estimation in learning was studied via an experimental control group approach at the grade 6 level using a unit on angular measurement. The estimation group was asked to guess the measure of each angle before measuring it, being given a 5 day teaching period. Test batteries included a general intellectual ability test and five other cognitive ability tests prior to the experiment, and a 54-item criterion test afterwards. Analysis showed:

i) significant correlations between estimating ability scores and general intellectual ability, number facility, spatial ability, and flexibility of perceptual closure

ii) groups taught to estimate had significantly higher scores for transfer, estimation and achievement than the control groups.

(d) A Study of the Relationship Between Estimation and Mathematical Problem-Solving Among Fifth Grade Students (Hall, 1976)

Sixty grade 5 students were chosen randomly from eleven classes, and placed in two groups of 30, an experimental and a control group. Each group contained 15 good 15 poor estimators (based on a pre-test). The experimental group received eight half-hour lessons in estimation and then both groups were tested in estimation and problem-solving. The following results were obtained

i) good estimators solved problems significantly better than

poor estimators;

ii) the students receiving estimation instructions (the experimental group estimated significantly better than the control group;

iii) estimation instruction made no significant difference to problem solving performance.

2. The Place of Estimation in The Mathematics Curriculum of the Junior High School (Skvarcius, 1973)

This was a contributed paper to a conference on Junior High School Mathematics and deals with i) the advantages of studying estimation; ii) necessary processes and skills involved; iii) appropriate ways of teaching estimation.

The single most important advantage is argued to be " as a most powerful procedure to check computational results." This includes the placing of the decimal point - a most important part of many algorithms. Secondly, estimation is an integral part of the division algorithm, and "it would seem reasonable to assume that the more competent a child is at estimating the better he will be in the use of the division algorithm. Skvarcius quotes J. D. Williams (1972) who pointed out the complexity and rigid detail of algorithms and the fact that the child is a passive learner. Estimation can reduce this passivity and the over-dependence on routine, and even reduce the complexity. Thirdly, there is a strong relatiorship between estimation and mental arithmetic, and a high percentage of adult

non-occupational uses of arithmetic are mental. Fourthly estimation can lead to the idea of upper and lower estimates, and hence to the basic idea of approximation, as well to inequalities. Finally,

> Estimating is in and of itself indispensable to the operation and planning of many eminently practical everyday projects. Budgets, tax rates, construction bids, merchandising, manufacturing and a host of many other activities are based on estimates; without intelligent estimates, they cannot be planned or executed. If students could be kept aware of this it should help to enrich their mathematical experience and deepend their insight.

Apart from reasonableness of answers, basic skills involved include rounding whole numbers and decimals, and multiplying and dividing by powers of ten. Both of these should be '<u>mental</u>' skills. More sophisticated estimation becomes possible with knowledge of the structural properties of number systems, and other devices. (such as factoring and skills of divisibility)

There are few suggestions available, Skvarcius claims, on how to teach estimation. He advocates integrating estimation deliberately into the mathematics curriculum, in particular, encouraging and ensuring guessing whenever possible. Finally, as already implied, practical and independent applications of estimation to such areas as business and science should be presented.

3. Mathematics and Approximation (G.A. Guilbaud)

This is an account of an address to the 3rd International Mathematics Education Congress in 1976, and deals with the concept of approximation from a mathematical point of view. (Guilbaud, 1976)

Guilbaud's central theme is that, contrary to popular prejudice, approximation should not be seen as an inferior part of mathematics, but as one of its strengths when approached in rigorous fashion.

"Instead of thinking like everybody:

'An approximation is something not exact', we should say, reversing the terms:

'What is an exact calculation? - A calculation for which no approximation is needed'.

And we could say, speaking with the linguists:

'Exact calculation is the "zero degree of approximation"'

Guilbaud sees "speaking rigorously of the approximate" as a challenge to man's intelligent activity. He examines briefly two practical areas in which the approximate is used. The first is the vernier scale, the second the construction of musical scales, and suggests that the systematic search for approximations might be used as a thread in a historical approach.

He leaves this idea, however, and instead addresses the question of the place of approximation in today's culture. Both in daily newspapers and scientific discussions we find that talking and thinking by means of phrases demanding approximation

are unavoidable.

Guilbaud calls this the first level of approximate thinking, and goes on to identify two further levels. In level two, some indication of the extent of the uncertainty is given, while the third level is reached when the idea of successive approximation is developed. In connection with real numbers, we have three phases (i) an approximate value (ii) an interval (iii) an indefinite sequence of intervals.

The discussion now become distinctly mathematical, and is extended to the need for control of the approximation in some way according to the purpose involved. He uses the mathematical concept of norms to generalise the discussion, and to extend it to a fourth probabilistic level, in which uncertainty is stated in terms of a confidence level.

He remarks somewhat pessimistically, that the teaching of probability has not made much progress, although the theory has. And so he ends philosophically by placing the development of ways of thinking in the context of changes from the time of Archimedes to that of Lebesgue in 1654. "The period since then is quite short." (Change takes time!)

4. National Assessment of Educational Progress

Two national assessments have been made in recent years the progress in school mathematics of children in the U.S.A. The first took place in 1972-73, with results finally published collectively in 1978 (NAEP I, 1978). The second took place during

1977-78, with overall results reported in 1981 (NAEP II, 1981).

(a) NAEP I

In an article on Estimation published in The Arithmetic Teacher of April 1976, by Carpenter and others the difficulties students, and even adults, have with estimation is illustrated by discussion of four estimation exercises. For estimates of the addition of four addends, 3 digit ones for 9 year olds and 6 or 7 digit ones for 17 years old and young adults, 31% and about 60% respectively got acceptable answers. The most prevalent error was failing to round the answer. At the 9 year old level, some 30% of youngsters could not recognise 30 as the number closest to 28 among 20, 25, and 35.

In research on estimation, Paull (1971) found students were inconsistent in their ability to estimate in different types of estimation, and also that ability to estimate correlated positively with solving problems by trial and error. Other studies by Ibe (1971) and Nelson 1966) found that training in estimation may yield gains in higher order processes like understanding.

The authors suggested the following points be considered in teaching estimation:

i) it should be integrated into the curriculum very early, continued throughout schooling, and applied wherever possible

ii) students need to be convinced that estimation saves time and effort when precise calculations are not necessary or possible. Hence, calculation should not be used to check the

estimate.

iii) estimation might be used with the hand-held calculator, as a check on the accuracy of its answer.

iv) students should have opportunities to decide when to estimate and how much error is tolerable.

v) estimating does not always allow strict rules to be followed, and students need some "qualitative instruction".

In the final report of the first assessment there is discussion of estimating and measuring length (p. 95), but only one specific problem was given on estimating the length of a line in inches. In this case, 39% of 9 year olds correctly estimated it.

b) NAEP II (1981)

In the 1977-78 report two quite different levels of estimation skills were assessed (NAEP II, p. 90). The first was using multiple choice questions involving selecting the closest measure to the height of a common object; the second involved estimating a length to the nearest unit (in or on). The results indicate that "few students at any level" (9, 13, 17 years old) have mastered basic estimation skills. In particular, in the exercise corresponding to the 1972-73 one, the percentage correct for the 9 year olds was 35%.

5. Applications of Elementary Mathematics (Crawford, 1979)

In a keynote address to the Fourth Annual Elementary

Mathematics Conference at S.U.N.Y., Buffalo, N.Y. which focussed on Applications, one section dealt with Approximation, Error and Accuracy. In this section, Crawford illustrates the concepts of significant figures, and the error and accuracy associated with a measurement. In particular, he shows these very clearly in tabular form (p. 8).

6. Teaching Estimation and Reasonableness of Results (Johnson, 1979)

Johnson begins this article in The Arithmetic Teacher by outlining the way in which reasonableness and estimation are currently taught in schools - in the first case after the fact, in the second emphasising primarily rounding in a mechanical "closed" type of teaching.

He then describes some teaching strategies for use in addition with whole numbers, first in estimation, then in reasonableness of results. In estimating, he argues for a three stage approach beginning with the leading digit for estimation (LDE) followed by the use of rounding and number sense as further refinements. He underlines the fact that the minimal skills needed are ability to do single-digit arithmetic and work with powers of 10.

In discussing reasonableness of results, he identifies two different types of reasonableness - physical and mathematical. In the case of physical reality, real-world results may be given and a rationale requested for their reasonableness or situations given in which something is to be estimated. For mathematical

reasonableness, knowledge of certain generalizations can be helpful in recognising errors, while with large numbers, estimation may need to be applied.

7. Identifying and Teaching Essential Mathematics Skills (I.T.E.M.S.)

The I.T.E.M.S. program was designed specifically to implement Recommendation 2 of the N.C.T.M.'s Agenda for Action. It was developed with U.S.A. Federal funds, and contains resource material for teaching 54 mathematical skills in grades k-8 with grade level diagnositc tests for each skill. Input from over 200 U.S.A., and Canadian mathematics educators was used as a basis for ordering the skills in priorty from 1 to 54.

The skills have also been placed in NCSM categories, so that in altertness to the reasonableness of results there are 3 skills, while in estimation and approximation there are 9 skills. Table I in Appendix B shows these 12 skills with the grade level in which they are developed in the program.

In the sample of materials provided, there are three activities related to estimation and three related to reasonableness of results. These are:

(i) grade K-2 - estimating distances by paces, followed by measuring and comparison of the closeness of the estimate (skill 10)

(ii) grades 3-5 - estimating whether $10.00 is enough to buy various sets of groceries (skill 13).

(iii) grade 6-8 - similar to (ii) except that five multiple choice questions dealing with other money situations such as buying gas and TV sets at greater difficulty levels are included (skill 13).

(iv) grades 3-5 - circle the most reasonable of three answers provided for each of 10 questions (skill 21).

(v) grades 6-8 - eighteen additions each with three addends and answers are provided. The student is to circle those with reasonable answers (skill 21).

(vi) grades 6-8 - three problem situations are presented in which the wrong operation or some other error occurs in obtaining the solution, and the student is asked to explain, and rework the problem (skill 22).

This seems an excellent program and merits more detailed examination.

8. Attitudes and Computational Estimation Skills of Preservice Elementary Teachers (Bestgen and others. J.R.M.E. March, 1980)

Research was carried out over a 12 week period using 187 preservice elementary teachers distributed over 3 treatments - control (T_1), practice in computation instruction (T_2), and practice complemented by instuction (T_3).

The purposes of the research were to:

1. characterize the feelings of preservice elementary teachers toward computational estimation skills;
2. collect benchmark data on the actual performance of pre-

service teachers on a variety of problems requiring rapid estimation;

3. study the effects of instructional lessons demonstrating techniques that can be used to estimate solutions to computational problems.

A number of measures were obtained both before and after the study. These included i) measures of attitudes to estimation, ii) measures of computational achievement and iii) measures of computational estimation. During the 12 week period, 10 instructional lessons were given to group T_3, focussing on estimation skills and strategies relevant to using the four fundamental operations. This group received 10 weekly quizzes, each consisting of 16 computation items, with brief discussion afterwards of 4 items recently taught.

According to the researchers, results indicated that

1. The preservice elementary teachers generally hold estimation skills in high regard, considering them necessary, good, and beneficial. Better estimators had favorable attitudes and perceived the process of estimation as more understandable and less complicated than did poorer estimators. The better estimators, however, did not feel that estimation was as necessary as did the poorer estimators.
2. Effects of instruction and regular practice activities on performance and attitudes were apparent. The group who received weekly quizzes showed significantly greater

achievements in estimation performance than did the group receiving no practice. Instruction on strategies accompanied with practice resulted in improved attitudes.

They concluded that this research documents the value of providing preservice elementary teachers with computational estimation activities. Brief systematic practice over a 10-week period increased performance on computational estimation activities. Furthermore, when such practice was accompanied with specific estimation strategies, greater understanding and respect for estimation processes occurred.

Details of only 1 instructional lesson (estimation techniques dealing with multiplication of decimal numbers p. 127) were given, so neither the sequences nor scope of the overall curriculum in estimation can be inferred from this report.

9. Teaching and Assessing Computational Estimation Skills (Reys and Bestgen, 1981)

This article first considers a definition of computational estimation, and then follows up by making suggestions for both teaching and testing it.

By considering a specific example, various elements involved in what computational estimation is are identified and the following definition offered.

"Computational estimation can thus be defined as an interaction of mental computation, number concepts, and technical arithmetic skills such as rounding and place

value."

To date, performance by students in this area has been disappointing (NAEP II and other results are cited). Lack of systematic instruction is clearly one reason, and four suggestions are made for teaching it. The first is "readiness discussion" aimed at developing good attitudes. Thus it is important that students see estimation not as a second rate but as "an essential and practical skill" to be distinguished from exact computational. Next, front-end strategy should be taught. Ways of doing this are outlined, as well as different follow-up activities. Thirdly, rounding as a technique is examined as it relates to front-end strategy, and partial work in mental computation with multiples of ten is considered important. Finally, an instructional sequence of i) mental development ii) brief practice iii) clarifying misunderstandings via discussion iv) regular review is recommended.

The last part of the article deals with testing, and factors such as time control, response format, and testing one question at a time are discussed. This is a useful section, with reference made to the need for a variety of structures, and for the inclusion of "complex" numbers which necessitate or encourage estimation being employed.

In conclusion the author stresses the need for better teacher awareness, more curricular development, and the construction of value testing instruments in order to facilitate the inclusion of systematic instruction in estimation.

10. Improving Computation - A Look at Place Value, Estimation and Checking as Significant Elements in Achieving Understanding and Correctness. Crawford, (April 1981)

During the course of surveying the literature the author prepared the above paper and presented it at the 8th Annual Conference of the Research Council for Diagnostic and Prescriptive Mathematics in April 1981. A modified version was subsequently accepted for inclusion in the proceedings which are now in press. Pages 3 through 22 relate to estimation and approximation and summarize some of the relevant literature. In particular, a flow chart was developed showing a possible network for developing these concepts in the curriculum. This is reproduced as Table 2. Some suggestions for an approach to teaching and learning concepts were also made.

11. Processes Used by Good Computational Estimators (Reys and others, J.R.M.E., May 1982)

This study was designed to identify and describe computational estimation processes used by good estimators and to characterize the thinking strategies and techniques they used when estimating. To accomplish this, a 55-item open ended test called the Assessing Computational Estimation Test was developed and administered to "over 1200 students in Grades 7 through 12 and selected adults". It included both straight and applied items with a reasonable balance between whole numbers and decimals.

Results from previous research, and pilot studies were used to assist in determining procedures. In particular, written explanations or interviews have typically been used to determine estimation strategies and studies relating intelligence with estimation or mental arithmetic abilities have tended to indicate that students with higher intelligence score higher on tests of estimation and use a greater variety of strategies than students with low intelligence. Thus the high school sample for study was chosen from upper-track and accelerated classes, and the adults chosen were successful in their occupation, and nearly all college graduates.

Each interview lasted about 40-60 minutes, and focussed on 5 straight and 10 applied computation exercises. Consistency among interviews was developed by "specific probes" and reliability was checked using a randomly chosen subject. The data base consisted of an Interview Summary Sheet, which used a classification system developed by the research team.

Three key processes were identified that seem closely associated with good estimation skills. They were:

1. Reformulation: The process of altering numerical data to produce a more mentally manageable form. This process leaves the structure of the problem intact.

Rounding, front-end use of numbers, and subtraction of "equivalent" numbers are examples of this.

2. Translation: The process of changing the mathematical structure of the problem to a more mentally manageable

form. This form was then used computationally to process the numerical data. Several different types of procedures were observed.

Using equivalent and/or differently ordered operations, are examples of this.

3. Compensation: Adjustments made to reflect numerical variation that came about as a result of translation or reformulation of the problem. These adjustments were typically a function of the amount of time available to make a response but were also influenced by the manageability of the numerical data, context of the problem, and the individual's tolerance for error.

Such adjustments may be made during, or at the end of mental computation, and may be either deliberate, or intuitive.

One strategy in particular, front-end estimation, was observed It took two main forms in particular - rounding, and truncating each of which was used at least two ways. The variety of strategies used by the interviewees with the same exercise is well illustrated by two examples - one on addition.

The researchers also posed the following two questions:

1) Are there specific qualities or traits associated with good estimators?
2) Do these good estimators call on specific skills and abilities that contribute to their estimation success?

The data suggested affirmative answers. The researchers identified a number of frequently occurring characteristics,

classified them as belonging to one of these dimensions - number skills, cognitive processes, and affective attributes and used this data to construct a 3-level model (p. 197) This is reproduced as table III. Further research is needed to test this model, but some classroom implications of it are suggested in another publication of Reys and Bestgen (1981).

12. Skill in Estimating Problems of Extent and Numerosity

(Siegel, Goldsmith and Madna. J.R.M.E. May 1982)

This research was designed (a) to assess developmental differences in children's estimation skills by collecting normative data on their performance on a variety of problems, (b) assess the validity of a proposed model of estimation on the basis of the children's performance, and (c) suggest modifications of the model, that is, specify problem dimensions that could guide further research.

The starting point was a "competence" model of the estimation process which distinguishes between benchmark estimation and decomposition/recomposition estimation. Benchmark estimation is essentially the application of a known standard whereas in D/R estimation, no benchmark is available, and decomposition must be applied to find one. This discovery is then followed by recomposition. Decomposition may be regular or irregular.

Twenty children, 10 boys and 10 girls, at each grade from 2 through 8 participated in the study. They were told about the project, and given a brief description of estimation, and made

estimate of 24 problems in a single session lasting about 40 minutes.

The results indicated the need to revise the model so as to represent the process of estimation as hierarchical. The initial "move" was sometimes affective rather than cognitive, and depended on whether the item to be estimated was large and the size of the available benchmarks.

Typically both numerosity and extent need to be estimated, so the model does not distinguish between them. Additionally, a factor needs to be included to allow for individual familiarity and ability with a variety of numerical concepts.

II.4 ESTIMATION AND THE HAND-HELD CALCULATOR

1. Three Relevant Research Studies (1976-1978)

(a) The Pocket Calculator: Its Effect on the Acquisition of Decimal Estimation Skills at Intermediate Grade Levels (Sutherlin, 1976)

Students at the same intermediate grade levels were exposed to a strong explicit emphasis on estimation as they studied decimal operations. For precise calculations, the experimental group used calculators, while the control group used paper and pencil. No significant differences in estimation skills were found when relevant test scores were compared, although gains were found as a result of the teaching of estimation. Among other conclusions, were that calculators provide incentive to attempt difficult problems, and support trial and error methods

of problem solving

(b) A Study of the Calculator's and Altered Calculator's Effect Upon Student Perception and Utilization of an Estimation Algorithm (Lawson 1977)

The major aim was "to assess what effect the hand calculator can have upon student ability and desire to use a standard estimation algorithm as part of a problem-solving strategy." Three groups of seventh graders were chosen randomly: a non-calculator, a regular calculator, and a an altered (operation keys masked) calculator group. Each group was given an estimation pre-test followed by a sixteen item computational task to be solved by paper and pencil, regular calculator, or altered calculator respectively. The same pre-test, reordered was then re-administered to all three groups. Results indicated:

i) computational ability correlates positively with ability to estimate

ii) students do not use the estimation process to verify calculated answers

iii) students recognize the disadvantage of becoming dependent on the calculator as the only method of obtaining an answer, yet trust its answers more than paper and pencil ones.

(c) A Study of the Effect of Hand-Calculators on Achievement, Estimation and Retention of Seventh and Eighth Graders on Decimals and Percent (Fesharaki, 1978)

Students from four mixed Grade 7 and 8 classes were categorized as high, middle or low achievers, pre-tested, and assigned in classes to one of two teachers. Each teacher taught two classes, one via calculators, the other traditionally for 30 class periods of 50 minutes each. Post test and an estimation test were then administered, followed by a retention test 4 weeks later.

Among the conclusions were the following:

i) The calculator method was more effective in estimation of mathematical skills than the traditional method.

ii) The experimental sub groups (H, M, L) achieved better with respect to estimation skills than their respective control sub-groups.

Further research using i) a large sample, and ii) other topics were recommended.

2. Bell (Arithmetic Teacher Nov 1976)

With the increasing availability of inexpensive hand-held calculators, the skill of estimating has assumed a new importance.

In a special issue of The Arithmetic Teacher devoted to calculators (Nov. 1976), Max Bell reports on explorations conducted in 20 classrooms and makes some interesting comments. In particular, he claims that high initial interest in calculators persists in children over a long time "provided students are given interesting things to do". The main difficulty, he claims,

is that "few existing school mathematics textbooks have really interesting problems that exploit the power given by calculations".

In answer to the question "Do children naturally detect errors and reject unreasonable results?", he answers that in their explorations they found that:

> children who have a good "number sense" can easily be led to use it in their calculator work. But most youngsters do not have that sure number sense and seem accustomed to accept whatever results they get, whether on paper or with a calculator. As to judgments about significant figures, nearly all youngsters merely write down whatever is on the calculator display, and often wish for even more digits. That is, the usual school neglect in teaching approximation skills and good sense about how many significant figures to retain is plainly revealed when children use calculators. Neglect of these matters must be corrected if calculators are used, but it should be corrected in any case.

3. Calculator Activities for the Classroom (1977)

This is a set of three books of dittomaster exercises, by Verwoort and Mason together with a Teacher's Resource Book and is a Canadian publication. Only Books 2 and 3 were available for scrutiny. Estimation receives a prominent place in each book covering pp. 27-48, pp. 34-67, and pp. 8-15 in each respectively.

"Estimating has always been an important life skill. The advent of the pocket calculator further increases the need for estimating ability. It is possible to arrive at an incorrect answer by pushing wrong buttons, pushing too many buttons, having a low battery, etc. Calculator users must develop a 'feeling' for what the answer should be, and critically examine answers that are displayed.

"Teachers should remember, however, that estimating ought to be a part of every mathematics lesson. 'First estimate, then calculate!"

Discussion of rounding is followed by the development of skills in estimting in connection with all four operations. Finally, in Books 2 and 3, it is applied with large numbers and in order to solve problems?

The style is informal with carbon-type suggestions and illustrations, and there are some interesting exercises, with sound help for the teacher in the resource book. e.g. shopping lists for gong camping (3:11) Bermuda Triangles (3:13) and problems involving large numbers such as those in (3:35). It is not stated what Grade levels these books are designed for, but they seem to cover the range for Grade 3 up to Grades 7 or 8.

4. Calculator Ideas for the Junior High Classroom
(Ockenga, 1976)

Several useful ideas relating the use of the calculator and the development of estimation and mental skills are suggested in

this article. They include:

i) the game of Check It which is an enjoyable way to practice basic facts

ii) a game using division exercises where players estimate first, have a referee determine the exact answer using the calculator, and then players score 0, 1 or 2 points according to the degree of accuracy of their estimate

iii) multiplying by 10 or 100 on the calculator, and finding it is easier to do it mentally.

5. Let's do It! Taking Advantage of the Hand Calculator (Bruni and Silverman, 1976)

This is an interesting article which includes two calculator games related to estimation and mental calculation. The first requires students to use three cards dealt to them to come close to the value of a key upturned card by adding. The 'estimates' are checked by calculator, and the closest is the winner (p. 449). The second requires three students each to solve a posed problem by a different method - mental calculation, pencil and paper, and using the calculator. Two purposes can be seen, one noted in the article.

"It can be very worthwhile to provide situations where children get the opportunity to use the calculator to help solve word problems. This can help them realize that, while the calculator is a tremendous tool, it does not help in solving a problem unless you know what numbers you want to

use, in what order, and what operations to perform." The other is to help children see that there are different ways - not just use of an algorithm - in which computation can be carried out.

6. Let's Start to Calculate (Davidson, 1976)

This is a book on Calculator Math for children in the middle grades (5-9). Four main uses of the calculator are seen:

i) checking computations
ii) helping pupils understand the decimal systems
iii) making vivid and meaningful the commutative and associative laws, and the
(iv) awakening interest in number theory for enrichment and recreational purposes.

The book is divided into 2 parts. Part I has 5 units designed to teach the operation of the calculator, the fifth of which deals with rounding and estimates.

There are four topics, and the general approach is to have students estimate and then check how close they are by using the calculator. Rounding is outlined as a necessary skill in estimating, and two of the topics relate to buying, one on addition, the other on multiplication. The style is relaxed and informal.

7. Calculation and Calculators (McHale and Witzke, 1977)

This book is written by two technical college professors and

is designed to teach a wide range of calculation skills using a scientific or slide-rule calculator with an algebraic-entry method. It is written in programmed format, based on a task analysis approach, and contains excellent sets of tests and answers.

The text is readable and clear, with a brief comment at the beginning of each section. There are 10 chapters in all, beginning with Elementary Calculator Operations. Of specific interest in relation to estimation and approximation are the discussion of rounding in Chapter 1, the discussion of scientific notation in Chapter 3, and the two final Chapters - Chapter 9 on Measurement Concepts and Chapter 10 on Estimation. In Chapter 9, clear explanation and practice is given in identifying and distinguishing between precision as the smallest unit of measurement used, and accuracy as relative error. Also included is a clear discussion of significant digits, and their relationship to accuracy (pp. 368-370). In Chapter 10, the emphasis is on well-defined procedures for estimating sums, differences, products, quotients, squares and square roots.

8. Scientific Analysis on the Pocket Calculator (Smith, 1977)

This is a very sophisticated, technical and comprehensive reference book for professional users of calculators. Its aim is "to supply the reader with a large number of numerical techniques, numerical approximations, tables, useful graphs, and flow charts for performing quick and accurate calculations with

pocket calculators."

Its main use is as a reference book for the high school mathematics or science teacher, and perhaps as enrichment study material for gifted students. It is extremely well written, and contains much excellent reference material and clear exposition of the nature of calculators and their application in many scientific fields.

9. The Calculator Work Box (Sharp, 1977)

This is a set of 80 work cards, arranged in 5 topic sections as follows: The Calculator (1-8); Number (9-32); Measurement (33-56); Real-Life Math (57-72); and Extra Points (73-80).

The first set enables exploration of the strengths and weaknesses of the minicalculator as a mathematical tool, focussing on the limitations of the calculator (overflow), and the behaviour of its built-in variables, together with some exercises on estimation using multiplication. No real rationale is provided for these, although they are useful exercises.

Several of the cards in the second set, (number 17, 18, 22, 23, and 26) deal with approximations, estimating and rounding. Number 17 and 18 are good illustrations of how the calculator can help to develop successive approximations once a first estimate for some answer, in this case 3, has been made. The skill of "rounding to a given decimal place" is assumed to be present. Number 22 gives good practice in expressing fractions as rounded decimals, and with significant figures while number 23 is similar

to number 17 and 18 but deals with cube roots. Numbers 25 and 26 deal with evaluating, again giving good practice in the use of the calculator as a computational tool.

These are useful and well designed routine types of exercises which could be made more interesting by giving more background of a historical and conceptual nature.

The Teacher's Guide gives very helpful notes on each of the 80 exercises and also useful tips on management, and evaluation. Every eighth assignment care is a Test Yourself card, and answers are provided in the teacher's guide.

10. Calculators in the Elementary Classroom (Drake, 1978)

This March 1978 Arithmetic Teacher article gives good suggestions on why and how to use the calculator in the classroom at the Grade 6 level for checking answers. Drake argues that checking gives a sense of accomplishment if no error is found, while it allows students to go over their own work or another student's if an error is found to be present. Thus, the use of the calculator acts i) as motivator ii) as a time-saver for the teacher. Children can also be helped to verbalize their errors.

Drake has also formed a Calculator Club, and has used it to focus on the function and methods of operation of the calculator.

> During such times we have discussed the human as a variable in the use of a mechanical device. If users do not have an idea of the answer they expect, then the calculator is of no value. We have also used this time to carefully

read the directions included with the individual calculators. Most often this is never done if the calculator is in the category of a Christmas toy. (p. 48)

11. Use of Calculators in the School Program - Interview with Gerald Rising (Ridge, 1979)

In an interview with Larry Ridge for the Ontario Mathematics Gazette, (March 1979) Rising made the following points:

i) the calculator only applies to about 10%-20% of the curriculum

ii) it can be used to help students develop basic concepts, e.g., the meaning of addition

iii) onus is on mathematics educators to identify the appropriate role for the calculator and develop curriculum materials

iv) three relevant areas in secondary school are graphing, logarithms, and numerical trigonometry.

12. Calculators: A Categorized Compilation of References (Suydam, 1979)

This is a systematic compilation of references related to hand-held calculators, carried out under a grant from the National Institute of Education and dated June 1979. The total list of references, nearly all annotated, comprises 155 pages, and there is an index giving references by topic. There are 4 listings under checking and 7 under estimation. Those judged

relevant and significant are discussed separately (see relevant research studies).

13. Calculators (Readings from The Arithmetic Teacher 1979)

This is an excellent sourcebook (edited by Bruce Burt) divided into four sections:

(i) Calculators: Policies, Uses, and Selection

(ii) Calculators and the Curriculum

(iii) Calculator Activities

(iv) Research.

Activities of particular relevance include an article by Bell and others reproduced from the 1977 N.C.T.M. Yearbook; a Mathematic Teacher article by Max Bell (1978); articles by Bruni and Silverman (A.T. Nov. 1976), Ockenga (A.T. Nov. 1976), and Litweller and Duncan (M.T. Nov. 1977) all of which include discussion of estimation.

The yearbook article first discusses four major reports which deal partly with calculators - the NACOME Report, the Euclid Conference report, an N.S.F. status study report by Marilyn Suydam, and the joint N.I.E./N.S.F. calculator conference report. Next, it discusses briefly a pot-pourri of calculator activities and experiments, and finally it lists some basic agreements.

As regards estimation, each report had positive recommendations to make.

14. Keystrokes - Calculator Activities for Young Students (Reys and others, 1979)

There are two books in this series, one on Addition and Subtraction, the other on Multiplication and Division. The publisher is Creative Publications Inc.,. Estimation is one of a number of activities which are included for use with a four-function calculator with algebraic logic and an eight-digit display.

The first exercises are on rounding of 2-digit numbers to the nearest ten, adding these 'estimates' and then using the calculator to check the estimate. There are a number of similar exercises in both addition and subtraction, with the additional idea of finding the difference between the estimate and the exact answer - also using the calculator. The numbers finally move to three-digits and there are some variations in the actual situation posed. Some of the situations seem somewhat artificial, but there is good experience being gained. The activities probably relate mainly to Grade 3, 4 and 5.

Games are also included, e.g., Match Me (p. 67) which depends on estimating difference successively between numbers and then using the calculator to check. Altogether, many situations which encourage, but not necessarily require estimation are presented. The style is explanatory, but still leaves a lot for the teachers to check on to make sure exactly what is being learnt.

The second book - Multiplication and Division - was not available for study.

15. Ditter and Mikesell (1980)

In their book entitled Teaching with the Hand-Held Calculator, Bitter and Mikesell claim that a unit on estimating is essential if students are to become really proficient users of the calculator.

> "The fear that students will become too trusting of the "gadget" is often expressed. It is true that students will get so they will take the answer that appears on the display as right and true if careful teaching of estimation is not accomplished." (p. 12)

The importance of rounding numbers and recognizing significant digits is also mentioned

> "often, students will report all 8 digits displayed (on the calculator) when just 2- or 3-place accuracy is warranted" (p. 15)

Included (p. 86) is one game involving multiplication of two-digit numbers which uses estimation, while Appendix A (pp. 301-306) provides a valuable resource for ensuring that three basic concepts on which facility in estimating depends are understood. The three concepts are:

(i) whole number and decimal place-value
(ii) rounding numbers
(iii) working rapidly and accurately with multiples are powers of 10.

Check-out tests for each of these three are given, together with re-teaching of the basic ideas involved.

16. Basic Mathematical Skills - Some Implications for Teacher

Education. Crawford (August 1980)

In a discussion paper prepared for a Teacher Education Conference at Athens, Georgia in August 1980, the author summarises recent literature from the U.S.A., Canada and England on the hand-held calculator (pp. 7-16). Of particular interest were questions raised by Blakeley (1978) in England. Among these were:

Are we clear about the difference between estimating and checking? Which should be used with a calculator?

Girling (1977) suggested that basic numeracy today means the ability to use a four-function electric calculator and discuss the importance of checking, understanding the relative size of numbers, and the role of mental arithmetic and algorithms as they relate to mathematical calculation.

Among sources cited which would merit further study are Neufeld (1978) and the 1979 Proceedings of the Canadian Mathematics Education Study Group.

17. A Mini Theme on Calculators (Ontario Mathematics Gazette, December 1980)

This issue contains eight articles on calculators. In one by Lesard, one example is given on ordering products by estimating. A more sophisticated article by Mura, Lagavine, and Muller considers Calculator Errors. This is a very useful source for understanding the approximate nature of calculator arithmetic. Also included are discussion of relative error (p. 18) and the dependence of the accuracy of a result on the accuracy of the

data (p. 20). Finally, an interesting article by Templeton introduces the idea of transcendental numbers, and discusses various approximations to ?? using the calculator, together with the amount of error involved by choosing particular approximations. This is a useful source for teacher education and enrichment activities for gifted students.

18. Calculators in the Classroom (Moursund, 1981)

This book was written with three purposes in mind - (i) to explain how calculators work, and what their capabilities and limitations are; (ii) to help elementary and middle school teachers apply calculators in classroom settings; (iii) to provide teachers with insights into the potential impact of calculators on the school curriculum. It is well organised, and contains many useful exercises and problems to which answers are provided. In addition, it contains four excellent appendices.

Of specific interest are (i) reference to the need for mental arithmetic to help catch calculator errors pp. 29-33, also (p. 121, 122) (ii) some discussion of developing mental computational skills (pp. 135, 136), and (iii) discussion of problems involving large numbers, which highlights the need to think in terms of significant digits, and includes good ideas for individual projects on estimating using large numbers (pp. 149-151).

19. BACpac - Basic Arithmetic with a Calculator (Watson, 1981)

This is a Keele Mathematical Education Publication by Frank Watson, supplied ready to cut up into 16 booklets produced for

use with 13-15 year old pupils of low mathematical attainment. Only very simple arithmetic is covered, with three aims in mind

(i) to give pupils some measure of success, via the calculator, in the hope of creating a more positive attitude to 'maths'

(ii) to allow them, undistracted by the difficulty of computation or their lack of accuracy, to glimpse the order and pattern which some of their more successful schoolmates are able to perceive

(iii) to give confidence and proficiency in basic mathematical processes without the calculator.

Included also are 10 games.

Items of particular interest are Booklet No. 5 called Estimates, and Game 9 (Estimate and Check). In Booklet 5, the student is helped to focus on numbers which are close, using the calculator to check. An estimate is defined as a number close to the right answer (in a whole number computation context). Game 9 consists of player A writing down two numbers with nearly the same digits, and player B trying to move from the smaller one which he sets in his calculator to the larger one in two moves within a time of 20 seconds.

20. Instruction in Estimating Solutions of Whole Number Computations (Schoen and others, JRME May 1981)

After stating five tentative generalisations culled from the literature on estimation, this article reports two related studies carried out at the fourth, fifth, and sixth grade levels.

In study 1, which took 13 days, one group of grade 4 students received five 45-minute periods of estimation instruction, and was compared with a control group on a number of measures.

The second study involved three treatment groups, drawn randomly from two Grade 5 and two Grade 6 classes. Insruction covered 5 days. T1 students completed 5 lessons on exact computation, T2 did similarly but in practicing items on estimation, not computation was required, while the T3 group used calculators together with worksheets on rounding and estimating. A battery of eight tests were used.

According to the Journal report of the studies, the results were consistent across the two studies and the researchers concluded that:

i) all methods of teaching estimation were effective

ii) there is no evidence of positive or negative interaction between treatments and exact computation skills

iii) there is some evidence of transfer from meaningful estimation instruction to estimation in verbal problems

iv) students receiving estimation instruction did adopt the valid estimation strategies they had been taught.

21. Twenty-Five Activities for Teaching computation with Calculators (Bestgen, Calculator Information Center Bulletin No. 11, Aug 1981)

Included are several activites involving estimation and/or mental arithmetic (Nos. 3, 10, 17, and 18). One (No. 10) is called mental computation game, and uses three dice one with numerals 1-6, a second with numerals 4-9, and a third with two+,

two-, and twox signs. The dice are rolled, and the answer computed mentally. If correct, the answer is entered on a calculator, and the first to 200 wins. A second (No. 17) lists a menu and poses question such as Betty bought 3 items and paid $1.47 - what did she buy?

This is a useful set of calculator activities, but no suggestions as to where to include them in the curriculum are given.

22. How to Develop Problem Solving Using a Calculator (Morris, 1981)

This is a booklet of 24 activities in tear-out page form published by the N.C.T.M. Three techniques using a basic four-function calculator are suggested. They are look for a pattern, make a chart or organised list, and guess and check. The intent is "to suggest an approach to integrate teaching problem-solving with calculators in the regular curriculum."

The second strategy "Guess and Check" stems from good ancestry, for Polya advocated the teaching of guessing. As the introduction states:

> "A guess is a starting point; the problem is begun. Too many students (and teachers) have the attitude that it is wrong to make a guess, perhaps because the method is often called "trial and error," and errors are to be avoided. However, by checking a guess, one can learn how to make a better guess--a process that can be repeated until the solution is found. This is the powerful strategy of making successive approxima-

tions. With a calculator to check guesses, this is often a very efficient way to solve problems."

Six of the activities relate to this technique or strategy (numbers 2, 6, 10, 14, 18 and 22). These six activities vary from guessing numbers (the first three) through successive approximation (number 14) to more demanding "optimum type" problems on money and shape respectively.

These are fairly straight forward, but powerful illustrations of what can be done to integrate the ideas of guessing and estimation with the on-going curriculum using the calculator, and are to be highly recommended. The specific activities seem aimed at about grade 3 to grade 7.

23. Calculators and Estimation (Goodman, A.T. Feb 1982)

This is a set of activities designed to give practice in estimation of very large and very small numbers. Three sheets of examples are provided. The stated objective is to estimate factors and discuss using a calculator to check estimations. While the exercises are interesting and well designed, they seem somewhat artificial, and the real value of them is not clear. It may be that working them through in a class would resolve this ambiguity.

II.5 ANALYSIS AND DISCUSSION

Analysis of the items comprising the survey showed that their subject matter related to one or more of the following categories - meaning and analysis of basic concepts; justification for including study of these concepts in the curriculum; research relating to estimation; examples of curriculum topics embodying the concepts and strategies for teaching or learning the topics.

The material derived from the survey was therefore examined in terms of these categories to see what the implications were for the mathematics curriculum of the 1980's.

1. The Meaning of Estimation, Approximation and Related Concepts

Estimation: From the dictionary definitions, estimate seems a somewhat loose term, in that it can be applied in both qualitative and quantitative situations - it may involve an opinion, or judgment, or calculation, and relate to people, things or numbers.

Among the sources surveyed, several attach rather more precise meanings to its use in mathematics. Estimation as a process implies a comparison between two things or sets of things, and implies that the estimator lacks total data to determine exactly the quality or characteristic he wishes to know. In school mathematics, we begin by studying number, shape and relationships related to them. Initially, the comparisons we make are crude "That man is taller than my father" or "There are more candies in your hand than in mine".

But basically, when we estimate we seem to imply number or numerosity in some way. According to Bright, the most common use of estimating is in measuring, e.g., "Estimate the width of this page to the nearest centimeter". Thus a number and a unit of measure are both involved. In this context, Bright (1976) defines estimating as follows:

> Estimating is the process of arriving at a measurement or measure without the aid of measuring tools. It is a mental process, though there are often visual or manipulative aspects to it. It requires that several ideas be firmly in mind: (1) the unit of measure to be used, (2) the size of that unit relative to familiar objects or to other units of measure for the same attribute, (3) other measurements in that unit, and (4) a commitment to perform the estimating so that the product is as close to the actual measurement as possible. Estimating is guessing, but the guessing must be educated. Wild guessing is not true estimating, except insofar as such behavior represents the first stages of the development of estimating skill.
>
> An estimate is the product of estimating. It is the description of an attribute in terms of either a number and a unit or a number alone.
>
> Two distinctions among terms must be clearly delineated. First, measuring and estimating are processes, whereas measurements, measures, and estimates are products of these processes. Second, physical tools are required for measuring

but not for estimating.

Siegel and others (1982) broaden Bright's (p.89) definition by including counting as well as measurement, but distinguish it from approximation.

> We consider estimation to be a process that is distinct from approximation. Whereas approximation provides a rough solution to arithmetic problems, starting and ending in numbers (e.g., 19 x 21 is about 400), estimation begins in the physical world and ends with a rough numerical solution. Approximation, however, is often involved as a component in complex estimation problems. (Siegel et al., p. 211)

Reys and Bestgen (1981) focus on the use of estimation in computation and conclude that:

> Computational estimation can thus be defined as an interaction of mental computation, number concepts, and technical arithmetic skills such as rounding and place value. It is a mental process which is performed quickly (without any recording tools) and which results in answers that are reasonably close to a correctly computed result.

Finally, Atweh (1982) links estimation with mental arithmetic as follows:

> Mental arithmetic is a method of thinking through a problem, performing an operation, or obtaining a result, as opposed to using paper and pencil or some other concrete aid. Estimation refers to one aspect of the result obtained, namely, its precision, as opposed to an exact answer or

solution. (p. 51)

Approximation: The dictionary definitions seem to agree that this word has a more restricted meaning 'being close to', 'nearly but not exactly correct' as opposed to 'roughly' in the case of estimation. In mathematics, it usually relates to a result - the end of some calculation or problem, and implies number.

Siegel (1982), already quoted, makes the distinction that approximation occurs only in the mathematical world of number. Payne and Sebert (p. 959) do not really distinguish explicitly between estimation and approximation but introduce the terms 'about' and approximation in connection with measuring the circumference of a circle. They prefer understanding of the concept to grow experientially.

> A physical conception of approximation and successive approximation arises with a measuring process and the use of smaller and smaller units of measurement. A formal notion of approximation grows from and brings new meaning to a student's work with estimations and measurements. (p. 208)

In work with the division algorithm, discussion can help to show that an approximation is a relation between two numbers, and not a property of a number itself.

Checking and Reasonableness of Results

When one has done a computation or problem, the question of its correctness or validity can arise. The process of testing this is often called checking. It implies an 'after the fact'

scrutiny of the computation or solution, not necessarily by a re-working of it, e.g., subtraction may be checked by addition. Neither is it necessarily foolproof as errors can also be made in the checking process.

Alertness to reasonableness of results seems both a less precise concept in terms of the process to be used, as well as permitting the procedure used to occur before, (during?) or after the attempt to complete the task.

To have the best of both worlds, it would seem logical to consider alertness to reasonableness of results as a strategy to apply before attempting the task, and checking to be its counterpart at the end of the task. Part of the checking procedure can of course be to ask again "Is the result reasonable?" The three objectives stated by the I.C.E.M.S. program relating to reasonableness, seem to focus both on prediction and after-the-fact checking, and merit further study.

Johnson (1979) has enlarged the discussion of reasonableness by identifying two types - the first puts reasonableness in a real-world context, the second in terms of mathematical relationships. He goes on to give illustrations of these, and how one might teach understanding of these.

The Impact of the Hand Calculator

With the emergence of the hand-calculator, a new dimension has arisen. The calculator itself is an approximate device, but only after a certain degree of complexity or precision is reached

or required. It throws into greater prominence all of the concepts just discussed. As Blakeley (1978) has queried: "Are we clear about the difference between estimating and checking? Which should be used with a calculator?"

More analysis of questions such as these based on both classroom experience and further research is clearly needed.

2. Rationale for Studying Estimation and Approximation at School

Why should the concepts of estimation and approximation be studied at school? There are many claimants for inclusion in the school curriculum, and so it is important to be able to justify items which are included.

Skvarcius (1973) lists four reasons for studying estimation. Briefly, they are:

i) estimation is a powerful procedure for ensuring the reasonableness of computational results

ii) estimation is an integral part of the division algorithm

iii) there is a strong relationship between estimation and mental arithmetic, and much of the common usage of mathematics by adults involves the need for good mental arithmetic skills

iv) estimation is an indispensable part of planning many everyday projects, whether government, business, industrial or personal.

Bright (1976) advanced a further reason by illustrating forcibly that estimation is essential for skilled measurement.

This reason, together with the more general claim that learning to estimate is an important component of problem-solving skills in general, are advanced by Siegel and others (1982).

Guilbaud (1976) examining the related, and more mathematically oriented concept of approximation, underscores the need to understand estimation and approximation in everyday life because in it one has to deal continually with figures of population, industrial output, percentages etc. In addition, he claims that the degree of precision or accuracy involved, and their limits require mathematical study and understanding in the world of production, quality control, and in many other aspects of industrial and social activity.

Similar reasons are put forward by Payne and Seber (1959) though more from the perspective of the relationship between mathematics and science, the need to distinguish "between mathematical conditions and the physical situations they describe" and the key understanding that all measurement is approximate.

Finally, numerous writers such as Bell (1976) Bitter and Mikesell (1980) and Moursund (1981), as well as the various recent reports and policy statement from N.C.T.M. etc., all point to the increased need for estimation and alertness to the reasonableness of results due to the heavy use of the hand calculator.

3. Research Relating to Estimation

Two major thrusts can be identified in the research studies surveyed. They are:

a) studies aimed at a better understanding of the nature of estimation, its relationship to other abilities, and the mental processes involved in using it

b) studies aimed at evaluating the effects of instruction in estimation on computational and other competencies.

A. Nature of Estimation and Processes Involved

Paull (1971) attempted to analyse the ability to estimate in computation and physical measurement by testing groups of student under statistically controlled conditions. He came to various conclusions, notably that there is a modest, but significant positive correlation between the ability to estimate and mathematical and verbal ability. Nelson's (1966) results support this conclusion. Schoen (1981) reported that a survey of the literature, by him in which he cited Kliebhan (1955) as well as Nelson and Paull yielded the following tentative generalization:

"The variables that appear to be related to estimation skill are general intelligence and mathematical ability. There is evidence that estimation skill is less strongly related to computational skill and verbal problem-solving ability, and there appears to be little relationship between the ability to estimate lengths or areas and the ability to do computations rapidly."

Research by Reys and others (1982) on the other hand first

identified "good computational estimators" by means of a test and then attempted to identify and analyse the processes they used to estimate. Three processes were found - reformulation; translation; and compensation - together with a number of other characteristics such as tolerance for error, knowledge of arithmetic properties and self-confidence. Each process and characteristic was assigned to one of three levels depending on the extent of its existence in the group of good estimators.

Finally Siegel and others (1982) tested out a 'competence' model of estimation based on rational task analysis which posited the two hierarchially related procedures of benchwork estimation and decomposition/recomposition, where decomposition takes place only if benchwork estimation fails to yield a suitable "reference unit" on which to base the estimation.

B. Instruction in Estimation

A number of the studies examined looked at the effects of instruction in estimation on related competencies. Nelson (1966) had an instruction period of one week at grade 4 and 6, Ibe (1971) used a similar period with the topic of angular measurement at grade 6, and Hall (1976) gave his experimental grade 5 group eight half-hour lessons. It is difficult to know how much weight to give to results based on such relatively short periods of instruction, and it seems highly desirable to carry out a more systematic set of studies to enable more reliable statements to be made about the effects of instruction. Schoen's

statements of his tentative generalizations are also couched in somewhat cautious terms.

Thus he states:

> The limited evidence indicates that teaching processes of estimation, at least the rounding process, improves the ability of fourth, fifth, and sixth graders to make good estimates. (Nelson, 1967; Sutherlin, 1977; Hall 1965)
>
> Three investigators (Newcomb, 1922; Wilson, 1922; Nelson 1967) presented evidence that instruction in estimation resulted in improved skill at solving verbal problems, but three others found no such gains (Washburne & Osborne, 1926; Dickey, 1934; Hall, 1965).

Use of the Hand Calculator

In an experiment on the effect of using the calculator for acquiring decimal estimation skills Sutherlan (1976) found no significant differences between groups using calculators and those using pencil and paper. Lawson (1977), in a study a Grade 7 level designed to identify the effect of calculator usage on the use of anf algorithm for estimation concluded that many students do not use the estimation process to verify calculated answers. On the other hand, ?????????? (1978) in a roughly similar study at grade 7 and 8 level using the topic of decimals and percent over a treatment period of 30 50-minute periods found that the "calculator method was more effective in estimation of mathematical skills than the traditional method".

Once again, it is difficult to make any reliable generalizations based on the evidence of these few studies. It is in fact debatable whether such studies serve a significant purpose. As Reys (1980) argues:

> "To consider calculator programs versus non calculator programs is to create another false dichotomy in mathematics education. Calculators must be viewed as another tool within the mathematics program and used as such. There are many topics appropriate for calculator usage and others equally inappropriate. These differences must be recognized and treated accordingly in developing instructional programs. Thus, the question is not should calculators be used, but when, where, and how can they be used most effectively."

Rising in the 1979 interview with Ridge made similar comments about the need to find the appropriate role for the calculator.

Again, the relationship between checking and the use of the calculator is not clean-cut as has already been pointed out by Blakeney (1978). Reys, in the same 1980 article just quoted questions the educational value of asking pupils to do lengthy calculations and then asking them to check their work with a calculator.

Clearly, we need more experimentation in the classroom, aided by continuing relfection and evaluation.

4. <u>The Curriculum in Estimation and Approximation</u>

What can we say about the place of estimation and

approximation in the school mathematics curriculum as a result of the literature survey?

First, two basic points should be made. It is generally agreed, e.g., Carpenter (1976), Reys and Bestgen (1981), that the study of these concepts should begin early and be integrated into the curriculum. Secondly, the relationship between their study and the use of the calculator needs to be developed both empirically and reflectively so that they complement each other. In particular, the meanings and applications of checking as one part of estimation need to be scrutinised carefully. [Blakeney (1978), Johnson (1979].

As regards the curriculum proper, a number of writers has made useful suggestions or proposals. These include those already cited, together with numerous others - in particular Payne and Seber (1959), Bright (1976), Trafton (1978), Crawford (1979 and 1981), Johnson (1979), Bitter and Mikesell (1980) and Reys and Bestgen (1981).

In addition, several of the books examined do in fact provide specific illustrations of sequences of instruction in estimation and/or approximation provided by their authors.

Without going into too much detail, and remembering that these individual authors can be consulted again for specific points, the following is offered as a synthesis of the literature, as interpreted by the present author. Reference can also be made to the network developed by him (Crawford 1981).

1. Early experience of situations in number and measurement

which involve 'about', 'near', 'close', initially qualitative and related to the real world, then moving towards more precision, and then the use of one digit and then two digit numbers.

e.g., Betty is about as tall as me.

Frank is a bit younger than me.

2. Exposure to real life situations where exactness may or may not be important.

e.g., What is your telephone number?

How far away from me do you live?

Discussion of situations like these (Reys & Bestgen 1981) can help to justify the value of estimating, [Carpenter (1976)] but should also help students to begin to understand the very basic difference between the exactness inherent in counting numbers compared with the unexactness or approximation inherent in measurement and things which are continuous as opposed to discrete.

3. Opportunities to estimate with marbles, beans, etc., should be provided with increasing attention to the more precise ideas of closeness and betweeness. 'Betweeness' affords a qualitative beginning to the idea of limits, and degree of approximation.

4. Rounding can now be introduced as a special procedure for obtaining an estimate. Front-end estimation has been supported as the beginning of a second stage process (Trafton 1978, Johnson 1979). The importance of the understanding of place-value including the concept of powers of ten should be obvious.

5. Continuing experience with estimation and approximation in the realm of measurement (length, distance, area, volume, mass, time and money) as suggested in the I.T.E.M.S. program and also by Bright (different kinds of estimation) should parallel this emphasis on rounding, moving gradually towards the use of decimal numbers, with common fractions not being forgotten.

6. The more sophisticated idea of the degree of approximation, together with the use of significant figures to indicate this in the way suggested by Crawford (1979) should be considered, leading gradually in both measurement, real world contexts and more mathematical contexts the related concepts of accuracy, precision, tolerance and error. Much of this work should be integrated with the study of other subjects, particularly science, and geography.

7. Finally, the use of estimation and approximation in high school areas of mathematics such as graphing, statistics and trigonometry should be an integral part of the study of these topics and the associated concepts.

Note: In all of this, care should be taken to decide when the use of the calculator is appropriate and to include discussion of relevant aspects such as the placing of decimal points, order of magnitude, scientific notation, and accuracy and error [see for example Mura in the Ontario Mathematics Gazette, Dec. 1980].

8. Teaching Strategies

Good teaching strategies are outlined in several of the books, yearbooks, and journal articles surveyed, mostly related to estimation - activities (Sharp 1977); Calculator Club, (Drake 1978), Problem-Solving (Morris 1981). More detailed suggestions on teaching estimation are to be found in Crawford (April 1981) and Reys and Bestgen (1981). Crawford outlines a sequence for developing the idea of an estimate for a 2 digit number carefully by moving from "more" and "less" through "how many more or less", using counters first, and then via a visual stage to the symbolic level. Reys and Bestgen give illustrations of a general instructional sequence consisting of i) mental development ii) brief practice iii) clarifying misunderstandings via discussion and iv) regular review. Finally, as regards approximation, an excellent sequence of learning the concept of approximation is developed in McHale and Witzke (1977) and background material abounds in the various yearbooks, especially those of 1959 and 1976.

III. THE SITUATION IN ONTARIO TODAY - QUESTIONNAIRE SURVEY AND DOCUMENT ANALYSIS

III.1 Introduction

The second part of the research study was designed to discover the present state of development in the school mathematics curriculum in Ontario of the concepts of estimation and approximation, together with the use of the hand-held calculator.

This was accomplished in two ways. First, a 4 part questionnaire dealing with this theme was prepared early in 1981 for completion by each of the 16 major boards belonging to Ontario School Mathematics Coordinators' Association. This was distributed initially in April 1981, and analysed initially during the summer of 1981, and finally in summer 1982. Secondly, relevant guidelines and documents from the Ministry of Education and the Boards involved, together with the most frequently used school mathematics texts as determined by Ministry Circular 14, 1980 and the relevant responses to the questionnaire were analysed.

III.2 The Questionnaire

(a) Preparation and Distribution

The 4-part questionnaire was prepared in February 1981. Part I requested general information relating to the process of curriculum development, the use of texts, and in-service education; Parts II and III were concerned with the present status in the system of estimation and approximation and the hand-held calculator respectively; Part IV requested information on future developments in these areas.

Originally, it had been hoped to make a presentation to the March 1981 meeting of the Ontario School Mathematics Coordinators' Association, but this proved impossible and had to be deferred until their May 8, 1981 meeting in Sarnia. In the meantime, a letter outlining in brief the research study and requesting copies of curricular materials relating to estimation,

approximation and the use of the hand-held calculator, was sent to each O.S.M.C.A. member. The letter also indicated that at the May 8 meeting copies of the questionnaire would be distributed, and asked the member's cooperation in completing it.

Copies of the questionnaire, the research study outline, and two covering letters, are to be found in the Appendix while Table gives a list of the member boards of the Coordinators' Association.

(b) Response to the Questionnaire and Request for Curriculum-Related Materials

Ten of the 16 coordinators were present at the May 8 meeting and received copies of the questionnaire personally. Of these, 7 subsequently completed the questionnaire. Questionnaires were mailed to the 6 absent coordinators, two of whom completed it, making a total of 9 completed questionnaires out of 16. Thus the actual percentage return was 56%. Documents and other curricular materials were received from most boards, although only four supplied materials relating to the area under study. Table 1 summarises this data, together with details of the geographical distribution and school populations involved. Since the total 1980 school population in Ontario was 1,815,084 the responses came from boards representing about 25% of all the school students in Ontario, and almost 60% of the students attending schools in the 16 boards which have membership in the O.S.M.C.A. The O.S.M.C.A. Boards are not representative of all Ontario Boards, in fact, they tend to be large boards (there are only 16

of them in a province which has some 60 Board's in all), and comprise the more wealthy and influential boards, which give leadership in school mathematics in Ontario.

Although a second personal appeal was made in 1981, no questionnaire was returned from Eastern Ontario, and so the data and responses obtained came largely from the Central Ontario region (7 boards representing 49.8% of the intended O.S.M.C.A. population, with the remaining 3 from Western Ontario.

(c) Analysis of the Questionnaire Responses

Tables with suitable headings corresponding to the questions asked were completed for each part of the questionnaire, then completed for each individual board responding. A final summary table was then compiled for all ten boards involved. Each board was allotted a code letter to preserve anonymity. The final result from the whole questionnaire was a set of tables with the following titles: Curriculum Development and Implementation; Use of Texts; Estimation and Approximation in the Curriculum; Use of the Hand-Held Calculator; Future Plans. The tables for the individual boards from which these summary tables were constructed have been retained for possible use by the boards themselves.

Part I A: Curriculum Development and Implementation

The questionnaire first requested general information concerning the nature of the process followed by the Board in developing the mathematics curriculum for the use of the classroom

teacher, as well as the problems or difficulties encountered in achieving this. All the responses received indicate that the process is initiated by the board itself and directed by its coordinators or consultants. (see Table) The early stages of the process of writing a curriculum appear to involve the production of a draft curriculum by the Board taking into consideration the provincial guidelines but based also on the needs of the students in the schools under the Board's jurisdiction. More than half the returned questionnaires refer specifically to the part played by teacher input and consultation at this stage. Although a number of the responses did not specify the expected time lapse between the writing of a draft curriculum and its final implementation in the classroom, the specific replies received suggested one or two years for the writing process and from one to five years for a pilot scheme or field trial before the completion of the process. The difficulties specified as having been perceived in the curriculum writing process appear to revolve around (i) the lack of time, money and (ii) the varying backgrounds of the teachers involved in implementing the mathematics curriculum at the classroom level. In the case of the larger boards, no difficulties were perceived.

The latter problem was tackled by providing some form of in-service education. All the completed questionnaires indicate that workshops are a part of their Board's in-service education directed towards assisting teachers to implement the curriculum. Although each Board indicates other activities

directed towards teacher education, such as P.A. sessions, department heads' meetings or the guidance of resource staff and consultants, the extent of in-service education and the expectations of the involvement of teachers vary enormously from one Board to another. Professional activity days used in this way vary from one to three per year, according to the questionnaire responses, and some Boards indicated that this kind of activity takes place only as requested rather than on a regular basis. Although many of the responses were not specific on this topic, it would appear that few Boards insist on a high degree of attendance at these sessions by their teachers.

Part I B: Use of Textbooks

7 of the 9 responding boards used commercial texts significantly in the Junior Division, with all 9 doing so at the Intermediate Level. Two boards used "home grown" texts at the Junior Level. There was about 50% moderate use of non-commercial texts overall.

The questionnaires also asked the respondents to list all commercial texts and research materials used significantly in the Junior and Intermediate Divisions. Four series were reported by six or more of the responding boards as being used. These were later analysed (see page , and Tables 12 through 15 in Appendix B). Numerous texts from other major publishers including new series were also cited. The list of approved texts in Circular 14 (1980) for most subjects in the curriculum is comparatively extensive. Selection of the texts to be used at the classroom

level is the responsibility of the individual boards or of the schools within their jurisdiction. This system leaves plenty of scope for flexibility in selecting texts to meet the individual needs of the students and to implement the curriculum that has been designed with those needs in mind. At least two of the responding boards have a permissive policy which allows schools to choose from Circular 14, or from a wide selection of Board recommended texts. The Ministry circular itself also suggests that the process of selection may fulfill another important purpose - "The involvement of teachers, department heads, and supervisory personnel in the selection of textbooks can....be a most effective means of in-service education."

Part II: Estimation and Approximation in the Curriculum

All the questionnaire responses indicated that the area of estimation and approximation is included in the mathematics curriculum drawn up by the responding Boards for both the Junior and Intermediate levels, but in most cases these concepts play only a minor role in the curriculum (see Table). Classroom resources related to this topic are on the whole devised by the teacher or generated by the Board consultants. In-service Education related to this area is apparently provided by seven of the Boards.

Within the area of estimation and aproximation the hand-held calculator is used only moderately in both measurement and computatio there being a good deal of variability.

Apparently Estimation and Approximation are also used quite extensively at the Junior and Intermediate Levels in other subjects, notably Science, and also in Geography, Social Science, Family Studies, Physical Education and Industrial Arts. This area would repay further study.

Part III: Use of the Hand-Held Calculator

The majority of the responding Boards indicated that the use of the hand-held calculator is included as part of their current curricula at either the 7 - 8 or 9 - 10 grade levels. Its widest use in the classroom revolves around problem-solving and checking of answers although as noted above it is used in a moderate way in some schools for computation and estimation. Boards have not, generally speaking, so far found it necessary or been willing to provide hand-held calculators for classroom use to anything more than a very minor degree. The extent of their use in the curriculum is thus seen as comparatively minor and is generally not associated with the use of commercial texts. Comparatively little direction appears to be given to classroom teachers in connection with the use of the hand-held calculator as part of the curriculum, either in the form of Board direction or in-service education, and its use in other subjects, with the exception of Science, is also very limited.

Part IV: Future Developments

While most Boards indicated that their future plans

include a more important role for concepts related to approximation and for the use of the hand-held calculator in the classroom, the responses do not suggest that a united front is being shown at the Board level in connection with these important areas. Several responses indicate that in connection with the use of the hand-held calculator there is a wide range of attitudes among the teachers in the classrooms. So far, the policy of the Boards has been comparatively laissez-faire as far as hand-held calculators is concerned and decisions concerning their use have been left very much to the individual teachers concerned. Hand-held calculators so far appear to have been viewed very largely as an extra, rather than as a useful tool that can play an integral role in the mathematics curriculum at the Junior and Intermediate Levels. It is unquestionable that in recent years the use of the hand-held calculator has increased without any action on the part of the Boards, and will probably continue to do so. A further complication is the emergence of the microcomputer. One Board commented that available money was being spent on the microcomputer. In most cases, it seems to be Board policy for students to provide their own calculators with prices now in the $10 range.

III 3. Analysis of Guidelines, Documents and Textbooks

As well as analyzing the questionnaire responses, various Ministry and Board Documents, together with four series of commercially provided textbooks were analyzed. A list of these

is provided in Table 10.

(a) Analysis of Ministry Guidelines

The guidelines published by the Ontario Ministry of Education in 1975 give direction in general terms to the part to be played by estimation in the school curriculum. The Formative Years suggests that the child in the Primary Division should "acquire competence " in the area of measurement in "estimating quantities by sight, touch, or by comparison with familiar objects... and confirm estimates by measurement." (The Formative Years, p.6.) In the Junior Division, greater prominence and therefore one would assume greater importance) appears to be attached to the ability to "estimate length, area, volume, mass, time, temperature and speed with an appropriate degree of precision." (The Formative Years, p.11.)

While the area of estimation and approximation is referrd to in similar general terms in connection with development of competence in measurement in both Education in the Primary and Junior Divisions (Ministry of Education, 1975) and the Intermediate Division Mathematics Curriculum Guidelines (Ministry of Education, 1980) it is not in itself suggested as an area to which any great emphasis should be placed in the school curriculum. The 1980 Intermediate Mathematics Guidelines includes the first indication from the Ministry that the hand-held calculator may be used as a tool "to introduce, reinforce, or illuminate new concepts" as well as to "apply estimation and rounding skills" and to investigate "approximation errors" (p.11). In the

curriculum outlines listed in the second half of this Ministry document greater emphasis appears to be attached to the area of estimation and approximation in connection with the Modified Mathematics programs than in the suggested programs for more able students.

Direction in connection with what happens in the classroom is also given at the Ministry level in the form of Circular 14, an annual publication which lists the texts approved by the Ministry for use in schools in Ontario. Those approved in 1980 are listed in Table VI. Six major publishers are listed, nearly all of them having math texts on the approved list at Primary, Junior and Intermediate Levels. Of the 32 entries, 24 are metric edition.s

(b) Study of Relevant Board Documents

Copies of various documents were received from most of the Boards being surveyed. They were of two kinds (a) policy documents relating to the use of hand-held calculators and (b) curriculum guidelines.

A. Use of the Hand-Held Calculator

Four documents on this theme were received, dating from 1976 to 1981. No doubt others exist. Only a few excerpts judged particularly relevant to the present study will be presented, together with some general observations and comments. Three of the documents were written in the period 1976 to 1978 at a time when calculator prices were dropping and dramatic increases in public availability of them were taking place. All agreed that

the calculator was not a panacea, but that "the important thing in mathematics are skills - what to do, when to do it, and a feeling of whether you are right or wrong."

> "If the student does not understand the concepts involved in a discipline, the calculator will be of no help at all."

The general thrust is well summed up by the following policy:

> "the judicious use of the calculator at various grade levels to support but not replace development of arithmetic skills is encouraged."

Many uses of the calculator are suggested and by 1980, the fourth document could state:

> "Correspondence from teachers using calculators indicates that slowly but surely these instruments are being assimilated into the instruction process particularly in the secondary schools. Whether or not to use calculators is not an issue. Most teachers and parents are accustomed to having these aids available."

No explicit mention is made of estimation in these documents, but the philosophy being declared seems to imply the use of estimati skills as part of the goal of learning to use the calculator intelligently.

B. Curriculum Guidelines

A study was made of curriculum guidelines provided by six Boards. The items related to the area of estimation, approximation and the calculator are summarised in Table XI.

Most deal with the Junior or Intermediate Grade, while one provides useful guidance for qualitative experience in estimating in the early grades. There are quite detailed suggestions in two of the most recent Board guides with an easy-to-follow format focussing on objectives, activities and teaching strategies and evaluation. Good illustrations are given, and these should clearly prove very useful to teachers.

(c) Analysis of Selected Series of Text Books

From the series of mathematics texts appearing on Circular 14, (1980) a list was compiled of those used by the ten responding boards. Four series were found which, according to the questionnaire responses, were used by at least 6 of them. These series were:

Junior Level — Mathways 4, 5, 6 (Copp Clarke, 1976)
Starting Points in Mathematics 4, 5, 6 (Ginn 1978, 1979)

Intermediate Level — Math Is/1, 2, 3, 4 (Nelson 1975-1979)
Mathematics 1, 2, 3, 4 (Holt 1976-1979)

For each of the four series, the Table of Contents and the Index were consulted to identify relevant material covered in the text. The topics identified are summarised in Tables 12 through

15. In general, there is little included in the use of the calculator. There is a fair amount of "standard" material on estimating related to computation, and occasional use of a realistic or interesting problem. Overall, the coverage is only moderate in amount.

III 4. Comments and Conclusions

The questionnaire survey and document analysis produced no really startling results. They were re-assuring in some ways, but not in others. For example, it was good to know that all the responding boards were including work on estimation and approximation from Grade 4 up to Grade 10, and that at least 6 of the 9 were using textbook series which covered the areas fairly effectively. Teacher consultation (6 out of 9) and the degree of emphasis on in-service education were less satisfactory (4 out of 9). Additionally, policy with respect to the hand held calculator seemed highly variable. The fact that the 1980 intermediate guidelines stress the use of calculators may well encourage considerable improvement in their use. There are available some good calculator resource materials and activities, including several produced in Canada, but there still seems to be lacking a true integration of this area into the curriculum on an ongoing planned basis, with a follow through from curriculum design to teaching materials and classroom activites backed up by on-going systematic teacher education. It seems too that with the impact of the microcomputer, consolidation of teacher and

pupil understanding of these important areas of the curriculum may be jeopardised. This remains to be seen.

IV SUMMING UP

The study has revealed a considerable body of literature on the concepts of estimation and approximation in the school mathematics curriculum in North America. According to research, it appears that estimation is related to general intelligence, and mathematical and verbal ability, that it is somewhat task-specific and that it does lead to improved computation. Attempts continue to be made to define it more clearly as a process and/or skill. The identification of the estimation processes of reformulation, translation and compensation by Reys and Bestgen and the procedures of benchmark estimation and decomposition/recomposition found by Siegel and others give some good bases for constructing curricular and teaching sequences. This can be supplemented by the skill analysis of the I.T.E.M.S. program, and the work of people such as Bakst, Payne and Seber and Bright using the kind of curricular framework previously outlined by Crawford.

Good sources for curricular material including work with calculators exist (e.g., McHale and Witzke, Moursund, Morris). More needs to be done, however, to widen the applications of these concepts to areas of real life. In particular, the everyday use of estimates and approximation, the significance of them in science and the changes which high technology is making in levels

of precision and accuracy all need continuing and intelligent research and attention.

All of these aspects need to be combined in a more concerted approach to educating teachers to the meanings and significance of the concepts of estimation and approximation and how they relate to the intelligent use of the calculator, and also of the computer. According to the survey of the situation in Ontario, the time is ripe for a sustained effort of this kind.

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APPENDIX A

BASIC DOCUMENTS

RELATING TO

THE QUESTIONNAIRE SURVEY

(i) Brief Outline of the Pilot Study

(ii) Covering Letter of February 25, 1981.

(iii) Questionnaire Distributed May 10, 1981.

(iv) Follow-up Letter of May 21, 1981.

Faculty of Education
Duncan McArthur Hall

Queen's University
Kingston, Canada
K7L 3N6

February 26, 1981

Attached please find a brief outline of a pilot research study which I am currently carrying out with a moderate grant from the Faculty of Education, Queen's University.

I would be greatly obliged if you could help me in the following ways:

(1) By sending me, as soon as possible, copies of curricular materials relating to the concepts of estimation, approximation etc. and the use of hand-held calcuatlors as used by your teachers in the school mathematics curriculum.

(2) By answering a questionnaire on the subject, which I will be preparing in the near future.

I have asked John Clark, your secretary, if he would put this item on the agenda of your March 10, 1981, meeting to allert you to my request. I had hoped to attend this meeting personally to meet with you individually, but unfortunately I have other prior commitments. It is my intention to be at the O.A.M.E. Sarnia meeting in early May, and I would greatly appreciate raising the matter then, distributing the questionnaire, and generally getting feedback on the project and its implications.

Thank you very much for any assistance you can give us.

Yours sincerely,

Douglas H. Crawford,
Professor of Mathematics and Education

A Brief Outline of A Pilot Study of the Significance of Estimation, Approximation and Related Concepts in the School Mathematics Curriculum *

by

Douglas H. Crawford,
Faculty of Education,
Queen's University
Kingston, Ontario

February, 1981.

* Conducted under a grant from the Faculty of Education
January-December, 1981.

Rationale

Recent thinking on basic skills in school mathematics in the U.S.A., the U.K. and Canada has led, among other things, to the identification of estimation and approximation as an area of high priority, especially in view of the emergence of the hand-held calculator as a readily available aid to anyone wishing and/or needing to carry out arithmetical calculations. Already a good deal of literature on the pros and cons of using calculators in the mathematics classroom has emerged, together with numerous experiments, including ones in Ontario (e.g. North York).

Generally, however, there has been a tendency for local boards in Ontario to approach this innovation cautiously. In view of the expectation that developments in this curricular area are imminent, and the recent launching by the N.C.T.M. of its An Agenda for Action - Recommendations for School Mathematics of the 1980's, the present author decided that a pilot study aimed at assessing the state of affairs in Ontario with respect to the concepts of estimation and approximation on the one hand, the use of the hand-held calculator on the other would be timely.

Purpose of the Study

The study aims

(1) to outline briefly the meaning and importance of the family of concepts stemming from the ideas of estimation, approximation, checking and precision.

(2) to survey the present place of such concepts in the curriculum by a study of present Ministry of Education guidelines, curricula currently used by selected Ontario Boards, and relevant mathematics texts and other documents.

(3) to relate the teaching and learning of these concepts to the availability of the hand-held calculator as an emerging tool for reducing computational labour and assisting in the better understanding of them.

Research Plan

The intention is to carry out the study in several interrelated phases.

1. Mathematical, scientific and lexicographical sources will be consulted in order to develo appropriate definitions and descriptions of the concepts involved, together with suitable illustrative examples. In this connection, a compucer search for relevant literature pertaining to the study will be conducted.
2. The Association of Mathematics Coordinators of Ontario, which is comprised of some thirteen or so coordinators representing major Boards of Education will be approached (i) as an association, and (ii) by correspondence and discussion with individual members, to ascertain their current thinking and practices. Relevant curricular guidelines and other documents will be collected and studied. Other Boards, (e.g. Frontenac may also be contacted (see 3, below).
3. Current Ministry of Education Mathematics Curriculum guidelines and other documents will be scrutinized, and the views and advice of relevant Ministry officials obtained as to other sources of data.

4. Mathematics textbooks authorized for use by the Ministry of Education, and other texts used by major Boards, together with any other relevant sources will be studied and anlyzed. In the first instance, the study will focus on the "middle school" curricular area, i.e. grades four through ten, and will be confined to the general area of arithmetic, measurement and geometry.

It is hoped to expand this study later to consider trends and developments in other Canadian provinces, the U.S.A. and Great Britain, and to integrate it with a more embracing long-term study of basic mathematical skills.

Expected Outcomes

(i) A better knowledge of the present thinking and practices with respect to these concepts and their place in the curriculum.

(ii) Written materials relating to these concepts and the hand-held calculator for subsequent use in curriculum development and teacher education, both pre-service and in-service.

PILOT STUDY ON ESTIMATION, APPROXIMATION AND USE OF CALCULATORS

by

D. H. CRAWFORD,

QUEEN'S UNIVERSITY

Questionnaire on Current Practices and Thinking in Major School Boards in Ontario

D.H. Crawford
May, 1981.

INTRODUCTION

The questionnaire is in the following four parts:

I. General Information.

II. Estimation and Approximation.

III. Use of the Hand-Held Calculator .

IV. The Future.

A fair amount of space has been provided for each answer. You may find it easier to append details of the answers to some questions on separate sheets, or by referring to your own documents which give the answer.

Please use the questionnaire only as a guide, and answer as freely as you wish.

PART I GENERAL INFORMATION

1. Outline briefly the process by which your mathematics curriculum is developed from the provincial guidelines. If the process varies according to Divisional level (Primary, Junior, Intermediate, Senior) please explain how.

2. Do you encounter difficulties in this process?

yes no

If so, please identify and describe them briefly.

3. To what extent do you use (a) commercial texts, (b) "home-grown" resource materials, (c) other resource materials in the Junior and Intermediate Divisions?

	significantly	moderately	only occasionally		significantly	moderately	only occasionally
Commercial Texts							
"Home-Grown" Resource Materials							
Other Resource Materials							

Additional Comments

4. List (separately if desired) all commercial texts and resource material used significantly in the Junior and Intermediate Divisions.

Additional Comments

5. Outline briefly what in-service education (nature, content, frequency) is (a) provided for, (b) expected of Junior and Intermediate Divisions teachers of mathematics.

(a) Provision

(b) Expectation

PART II ESTIMATION AND APPROXIMATION

1. Does your current curriculum include learning about estimation and approximation?

yes no

2. If so, give details by checking items appropriately below and making additional comments.

(a) Grade Level(s) 4 5 6 7 8 9 10

(b) Focus and Use of Hand-Held Calculator: Measurement (with / without); Computation (with / without)

(c) Extent: Minor Moderate Major

(d) Source: Commercial Texts; Resource Materials Own; Other

(e) Provision of Education: Yes No

(f) Additional Comments

3. Are estimation and approximation used in other subject areas at the Junior and Intermerdiate Division levels?

yes no

4. If so, give details.

Any Further Comments

PART III USE OF THE HAND-HELD CALCULATOR

1. Does your current curriculum include the use of the hand-held calculator?

yes no

2. If so, are calculators provided by the Board? How, and to what extent?

3. If used in the curriculum, give details by checking items appropriately below and making additional comments.

(a) Grade Level(s)

4 5 6 7 8 9 10

(b) Purpose

Aid in regular computations Estimating Checking

Concept Development Problem Solving Other (explain)

(c) Extent

Minor Moderate Major

(d) Source

Commercial Texts Resource own Materials others

(e) Provision of In-service Education

Yes No

(f) Additional Comments

4. Is the hand-held calculator used in other subject areas at the Junior and Intermediate levels?

yes no

5. If so give details.

Any Further Comments

PART IV THE FUTURE

1. What plans (with time-line if possible) have you for the future for the inclusion of estimation, approximation and the hand-held calculator in the cirriculum? (Use with statistics/probability, real-life applications, cooperation with other subject areas, etc.)

2. Describe any problems or difficulties you foresee (attitudes of teachers;, parents; cost; availablility of in-service education etc.)

Faculty of Education
Duncan McArthur Hall

Queen's University
Kingston, Canada
K7L 3N6

May 21, 1981.

Dear

Unfortunately you were not present at the May 8 Sarnia meeting of the provincial group of mathematics coordinators which I attended, and at which I outlined and distributed a questionnaire relating to my research project on Estimation, Approximation and the Use of the Hand-Held Calculator.

I am therefore enclosing a copy of the questionnaire, together with a stamped, addressed envelope, and would greatly appreciate it if you could respond to the questionnaire as soon as possible, and in any event before the end of June.

The questionnaire is restricted to the use of calculators, but if you wish to link the issues and developments connected with this, to the oncoming arrival of microprocessors, I would be delighted to hear your views and comments.

Finally, I am anxious to have copies of Board Documents, such as proposals to the Board, Position Papers and relevant Curricular Guides as a further source on which to draw in assessing the situation in Ontario with respect to these important curricular issues.

Thank you very much for any assistance you can give me in this project. I shall, for my part, keep you informed of the progress of this study, and its final conclusions and recommendations.

Yours sincerely

Douglas H. Crawford

3. Any final comments and/or suggestions.

Thanks very much for your cooperation. I hope to keep in touch with you and will send you a summary of the results of the questionnaire after it has been analyzed.

D. H. Crawford

May, 1981.

P.S. Please send the relevant curricular or other materials if possible.

APPENDIX B

TABLES 1 THROUGH 15

Table 1

Relevant I.T.E.M.S. Skills List

Placed in NCSM Categories

B. Alertness to the Reasonableness of Results

1. know by the unreasonableness of an answer that the wrong operation or wrong number was used (22)
2. be able to predict, with no more than 10% variance, the answer to a one or two step problem (54)
3. will realize from the data in the problem that an answer could not be a solution to the problem (21)

C. Estimation and Approximation

1. estimate size & weight (16)
2. estimate length & distance (10)
3. estimate area & volume (24)
4. estimate ratio & proportion (43)
5. estimate markup & discounts (25)
6. estimate the passing of time (11)
7. estimate percents (28)
8. estimate money totals (13)
9. estimate commissions (32)

Note: Figures in brackets indicate priority level assigned to each skill.

Table 2

A Possible Network for Developing the Concepts of Estimation and Approximation*

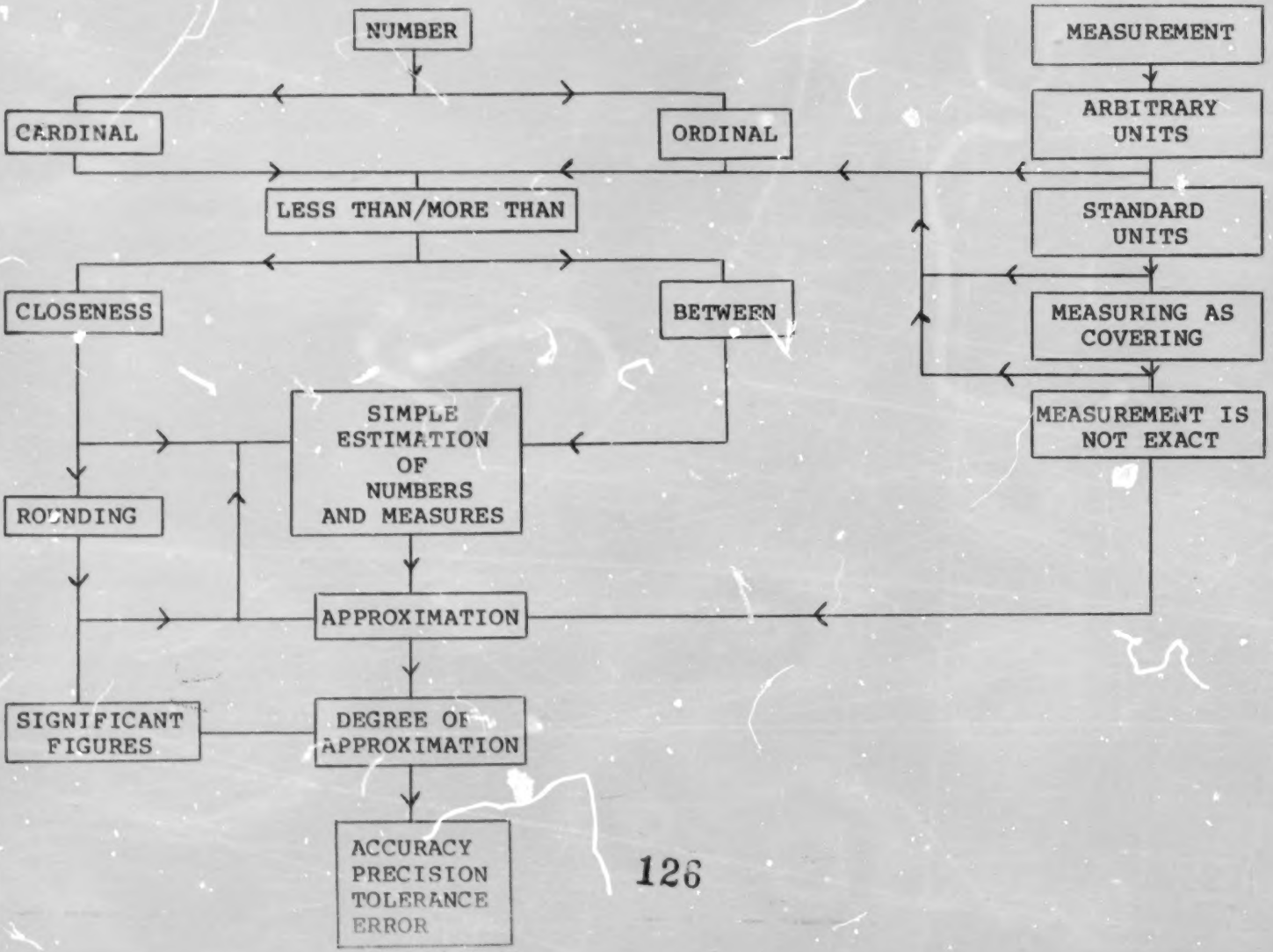

Table 3
Identifiable Characteristics Associated
With Good Estimators*

Level 3 20-50%	Mental Computation (with all types of numbers)	Intermediate Compensation	Variety of Strategies	Self-Confidence	
Level 2 >50% majority	Reformulation (rounding to manageable or compatible numbers)	Final Compensation	Translation	Arithmetic Properties	
Level 1 everyone	Basic Facts	Place Value	Reformulation (rounding to multiples of 10)	Mental Computation (involving rounded numbers	Tolerance for Error

Level 1 - each characteristic present among every person interviewed
Level 2 - each characteristic present among a majority of persons interviewed
Level 3 - each characteristic present among 20 to 50% of the persons interviewed

Denotes key processes

Reformulation

Translation

Compensation

*Extracted from Reys, R. and others, p. 197, J.R.M.E. May 1982.

BOARD	Curriculum Development: Process: Process initiated by Board	Process directed by Coordinators	Teacher consultation and input	Needs assessment -- draft curriculum	Provincial guidelines -- draft curriculum	Field trial or pilot scheme for draft curr.	Time line	Time to produce curriculum not specified	Guidance from Board for teacher implementation	Problems/Difficulties: Time	Money	Teacher attitudes	Varying background of teachers	Lack of resources	Inservice Education: Nature: Workshops	P.A. sessions	Heads of department meetings	Resource staff give in service guidance	Extent: On regular basis	As requested	No. of P.A. days	After School sessions	Expectation: All teachers attend	Some teachers attend	Not specified
A	✓	✓	✓	✓		5yrs ✓	1yr		✓	✓					✓	✓					3		✓		
B	✓	✓	✓	✓	✓	1yr ✓	2yr ?			No Difficulties					✓			✓		✓					
C	✓	✓		✓				✓	✓	✓	✓		✓	✓	✓	✓	✓		✓		1		✓		
D	✓	✓						✓					✓		✓		✓		✓		3				✓
E	✓	✓			✓	2yr ✓	2yrs +			No Difficulties					✓	✓					2 per ann			✓	
F	✓	✓	✓			✓	1yr	✓		✓					✓			✓		✓					✓
G	✓	✓	✓					✓		✓		✓			✓	✓				✓	1 per ann				✓
H	✓	✓	✓		✓				✓	NONE						✓	✓	✓	?	?	?	✓			N/A
I	✓	✓	✓						✓	✓	✓	✓	✓		✓	✓		✓	✓		8		✓		
TOTALS	9	9	6	3	3	4	3	4	4						8	6	3	4	3	3			3		3

Results of Questionnaire

Table 5

Part (b)

Use of Texts

BOARD	Junior Division									Intermediate Division								
	Commercial			Home Grown			Other			Commercial			Home Grown			Other		
	SIGNIFICANTLY	MODERATELY	OCCASIONALLY	SIGNIFICANTLY	MODERATELY	OCCASIONALLY	SIGNIFICANTLY	MODERATELY	OCCASIONALLY	SIGNIFICANTLY	MODERATELY	OCCASIONALLY	SIGNIFICANTLY	MODERATELY	OCCASIONALLY	SIGNIFICANTLY	MODERATELY	OCCASIONALLY
A		✓		✓				✓		✓				✓			✓	
B		✓		✓			✓			✓				✓				✓
C	✓				✓		✓			✓				✓				✓
D	✓					✓			✓	✓								
E	✓				✓				✓	✓				✓				✓
F	✓					✓			✓	✓					✓			✓
G	✓					✓			✓	✓					✓			✓
H	✓					✓		✓		✓			✓				✓	
I	✓					✓		✓		✓				✓			✓	
TOTALS	7	2		2	2	5	2	3	4	9	-	-	1	4	3	-	4	4

Table 6
Results of Questionnaire
Part II - Estimation & Approximation

Board	Curriculum Included in Grade Levels 4-6	7-10	Focus & Use of H. H. Calculator Measurement With	W/O	Computation With	W/O	EXTENT OF USE Minor	Moder	Major	SOURCE OF MATERIALS Comm	Own	Other	IN-SERVICE EDUC.	USE IN OTHER SUBJECTS
A	✓	✓		✓		✓	✓				✓		✓	Soc. Science Geog/History Science
B	✓	✓	✓		✓			✓			✓		✓	----------
C	✓	✓		✓		✓	✓						No	Science
D	✓	✓	✓			✓		✓		✓	✓	✓	✓	No
E	✓	✓	✓				✓				✓		✓	Science
F	✓	✓		✓	✓			✓		✓		✓	✓ for Gr. 7-8 teachers	Science Phys. Ed.
G	✓	✓		✓		✓	✓						No	Science
H	✓	✓		✓	✓		✓	✓		✓	✓	✓	✓	Science
I	✓	✓		✓	✓			✓	✓	✓	✓	✓	✓	Science Geography Phys. Ed.
TOTALS	9	9	3	6	4	4	5	5		4	6		7	

Table 7
Results of Questionnaire
Part III - Use of Hand-Held Calculator

Board	Use	Provision By Board	Grade Levels 4-6	7-8	9-10	Purposes Com	Est	Che	Con	P/S	Other	Extent Mi	Mo	Ma	C	Source Own	Other	In-Serv. Ed.	Other Subjects
A	Yes	No			Yes			✓		✓			✓			✓		No	No
B	✓	No		Yes				Extra Activity					Yes			Yes		Yes	Science Tech.
C	No	Planned in Future																	No
D	Not Spec.	No					✓	✓		✓							✓	Yes	No
E	Yes	7-8 9-10 No		✓	✓	✓ Calculation rather than concept development				✓		✓				✓		✓	No
F	Yes	15 per needy school		✓		✓	✓	✓	✓	✓			✓		✓	✓		✓	Science
G	No	1 class set can be borrowed from Board				No specific Board policy on use of calculations - up to individual teachers						✓							Science Geog.
H	Yes	Yes		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓		As required	Various subj. dep. on schools
I	Yes	No			✓							No response							Science Bus. Ed.

Table 8
Results of Questionnaire
Part IV - Future Plans

Board	Plans for Future Use					Difficulties					Final Comments
	Stats Prob	Real Life	Other Subj. Areas	Other	None	Teach Attit	Parent Attit	Cost	Avail. of In. Serv. Educ.	Other	
A	✓	✓	✓					✓	micro comp.		
B	No direction at present										
C	✓	✓			✓	✓		✓			Calculators regarded as an extra, not as an integral part of the curriculum
D	Pilot exp. in use of calculator for students in Grades 4-6										Wide range of attitudes to use of hand-held calculator in classroom
E				✓		✓					More widespread use and acceptance anticipated after 2 year field trial
F				✓		✓					1. More emphasis on estimation in future. 2. Curriculum being revised grades 1-8 3. Resource materials for hand-held calculator being developed.
G					✓						1. Use of hand-held calculator has increased without action on part of the board. 2. Board money going on microcomputers not calculators.
H	UNDECIDED					UNAVAILABILITY OF IN-SERVICE EDUCATION					----------
I	No	✓		✓		✓	✓	✓			----------

TABLE 9

List of Data Sources

Used in Document Analysis

1.	Ontario Ministry of Education	
	The Formative Years	1975
	Education in the Primary and Junior Division	1975
	Mathematics Curriculum Guidelines for the Intermediate Division	1980
	Circular 14	1980
2.	Board Documents	
	Policy documents on Hand-Held Calculator	(1976-1981)
	Curriculum Guidelines	(1978-1981)
3.	Commercial Textbook Series	
	Mathways 4,5,6 (Copp Clark)	1976
	Starting Points in Mathematics (Ginn)	1978,1979
	Math Is/1,2,3,4 (Nelson)	1975-1979
	Mathematics 1,2,3,4 (Holt)	1976-1979

Table 10

Major Textbook Series Listed in Circular 14, 1980

GRADE	PUBLISHER					
	HOLT	COPP CLARK	GAGE	GINN	NELSON	RYERSON
Primary	✓	✓	✓	✓	✓	
Junior	✓	✓	✓	✓	✓	
7	✓	✓	✓	✓	✓	
8	✓	✓	✓	1975*	✓	
9	✓	✓	Math 9* 1966	✓	✓	✓
10	✓	✓	1967*	✓	✓	✓

* - indicates non-metric text

Note: Earliest metric text listed in Circular 14 was Gage - Math Book 1 (Grade 7) 1974.

Table 11
Summary of Relevant Topics from Board Curriculum Guides

Board Code Document and Data	Grade Level	Summary of Topics
(A) K-6 (1980)	5	Rounds numbers to nearest ten, hundred, thousand.
	6	Extend rounding to nearest million for purpose of estimation and comparison.
Intermediate (7-10) (1981)		This is a 3 column document with headings objectives/ content; typical questions; and teaching notes and strategies.
	7	Check reasonableness of result by estimating answers to computations for all four operations (whole numbers, then decimal numbers); Rounding to nearest 10, 100, 1000, 100000 and to nearest 0.1, 0.01, 0.001. Understand usefulness of an approximate number. Estimate areas of simple closed figures in sq. cm.
	8	Continuation of Grade 7 work with inclusion of mental computation. Find square roots using estimating and rounding off.
	9 Advanced	Use estimation to avoid calculator errors in 'large number' computation.
	9 General	Main decimal approximations for square roots, perhaps with calculator.
	10 General	Round off whole numbers and decimals, and apply to real life situations. Approximate and estimate results. e.g., sales of football tickets, car ownership, grocery budget buying.
(B) Core (1980)	7	New Grade 7 Math Core Curriculum is first part of a 4 phase development so that by 1983-84 all grades 7-10 will have fully implemented curriculum. Document has a 3 column format-objectives, activities, evaluation. It is in 3 sections: Numeral Methods (pp. 1-17); Algebra (pp. 18-20); and Geometry (pp. 21-28). Calculator activities are suggested for use with nearly all of the 47 Numeral Methods Objectives - several objectives relate to estimation (e.g., N5, N7, N8), and suggestions are given for related calculator activities. e.g., estimate number of books needed to cover a CFL football field.

(E) Junior (1970)		The guide has a 3 column format of behavioural objectives, comments and vocabulary.
	4	Estimating area, and checking by measurement, and in problem-solving. Rounding to nearest ten, hundred, thousand.
	5	Review and extension of rounding. Estimation and Approximation in the four operations, and of length, area. Use in problem-solving to determine reasonableness of solution.
	6	Continuation of Grade 5 work. Rounding with decimal numbers. Estimating areas and volumes of squares, cubes, triangles etc.
Guideline (Revised) 1980	7	Estimate, Calculate, Check for Reasonableness, Rounding each number to one significant digit in decimal problems. Estimate distances, area of regular, irregular figures; volumes of rectangular solids by counting unit cubes.
Guideline (Revised 1980)	8	Estimate in real life problems. Investigate rounding errors resulting from successive calculations, especially with calculators, e.g., find difference between 73.65 x 8.64 rounded to the nearest tenth before and after multiplying. Round fractions and mixed numbers to the nearest whole number.
(F) 1-6 (undated)	Level 1	Comparing length and height without measuring. Relating object length to a standard length of 1 metre. Estimating and finding number of containers needed to fill a large container.
	Level 2	As in level 1 only more emphasis on measuring after estimating.
	Level 3	Much experience in estimating and measuring in i) litres ii) millilitres.
	Level 4	Estimate a length by drawing, then measuring. Estimate product (3d by 1d; 3d by 2d).
	Level 5	Understand shifting of decimal point by approximating results. e.g., 29.56 x 100 = 3000 (multiplication and division).
	Level 6	Continue work of level 5. Check reasonableness of quotient using rounding.

Table 11

(H) K-6 (c. 1979)		Calculators are not part of the K-6 program, but since they are at the Intermediate Level, a firm foundation of basic arithmetic skills including "many opportunities to estimate, approximate, round-off numbers and use memtal arithmetic" is essential.
7-10		Use of calculators is strongly recommended when problem-solving or incidental or secondary use of computation is involved. A well-sequenced illustration of using estimation followed by computation is provided where the problem is finding the area of a brick wall.
(K) Core Grade 7, 8 (1979)		Estimate sums, differences, products, quotients with whole numbers by rounding. Use calculator to estimate values of large powers, e.g., find closest estimate for 12^9. Rounding decimal numbers, and determining maximum rounding error. Estimating using all four operations with decimals. N.B. Similar topics, but with more difficult questions and situations in Grade 8.

Table 12

Textual Material on Estimation, Approximation and the Use of the Hand-Held Calculator
Mathways 4, 5, 6 (Junior) - Copp Clark

Book	Year of Publication	Grade	Topic and/or Page Reference	Summary and Comments
4	1976	4		No listings of expected topics in index. Mathematics Dictionary p. 304
			Mathematics Dictionary (p. 304)	Estimate: A calculation usually made by rounding off numbers to a multiple of a power of 10. 9 x 31 is about 10 x 30 or 300
			Calculator	No listings in index.
5	1976	5	Rounding Numbers pp. 238, 239	Rounding to the nearest Unit and nearest tenth.
			Application to Division and Problems pp. 240-243	Application to problems like "In a 14 hour flight the jet used 143700 Kg. of fuel. About how much fuel did it consume each hour?
			Math Dictionary (p. 340)	Estimate (as in Book 4)
			Calculator	No listings in index.
6	1976	6	Rounding Numbers to the nearest tenth pp. 96-102 176-177 319-320	As in Book 5 for use in 96-177 division 176-177 Review of computations and related problems 319-320 Use in rounding percentages in circle graphs
			Math Dictionary (p. 352)	Estimate - (as in Book 4 and 5)
			Calculator	No listing in index.

Table 13
Textual Material on Estimation, Approximation and the Use of the Hand-Held Calculator

Starting Points in Mathematics, 4, 5, 6 (Junior) - Ginn and Co.

Book	Year of Publication	Grade	Topic and/or Page Reference	Summary and Comments
4	1978	4	Estimating Sums (ii) 46-47 (iv) 200	To the nearest hundred and thousand, with estimate first then computation; whole numbers then to nearest unit with decimals.
			Estimating Product 201 234-234	Multiplication by single digit whole number of small one decimal place number. Extension to nearest ten and nearest hundred (whole numbers only)
			Estimating Applications length (cm, dm, km, m, mm) 204-211 280-281	Charts and examples are used to give experience in estimating, measuring and choosing suitable measuring units.
			concept 290-291	Discussion of idea of estimate and how to use it in given situations, e.g., how many eye blinks in an hour.
			Rounding numbers (i) 14-15 (iii) 198-199	Explains rounding to nearest 1000, 100, 10. Rounding decimal tenths to whole numbers.

5 (Ginn)	1979	5	Estimating sums 24-26 differences 36-37 product 60-152 quotient 208-209	to nearest 1000, 10,000. to nearest 100, 10, this unit with decimals in multiplication.
			capacity l,ml 222,223	uses real objects, but only pictures shown.
			mass g, kg 226,227	similar to capacity work.
			Rounding numerous 10, 11 references to 208-209	moves from rounding to nearest 1,000,000 through nearest unit and tenth in addition, subtraction, multiplication and division: whole numbers, then decimals. Useful and interesting problem situations.
			Calculator	Numerous references. Emphasis is on learning which keys do what, idea of floating point.
			p. 213	Sequence of operations is related to mental arithmetic
			p. 254	Importance of rounding and estimating is emphasized to ensure correct keying.

6	1979	6	Calculator 32-33 78-79 153 242-243	 Addition and subtraction Use of keychart Use of decimal point on display Fractions, also idea of doing mental calculation first is implie e.g., what will display show after 3 x 1 3 =
			Estimating sums, differences, products, quotients	Similar to Book 5 with use of rounding with decimal factors, dividends, and quotients.
			length 106, 107 area 112-113	similar to Book 5 idea of lower and upper units by counting squares
			capacity 160-161 l, ml	similar but more refined measures and cylindrical shapes
			mass 162-163 g, kg	uses various formats to obtain finer measures and related estimates
			Rounding Numerous References	 Extension of Book 5 Application to real life situations, e.g., rounding decimals in money, all 4 operations.

Table 14
Textual Material on Estimation, Approximation and Use of the Hand-Held Calculator in Selected Mathematics
Nelson. Math Is/Books 1, 2, 3, 4 (Intermediate)

Book	Year of Publication	Grade	Topic and/or Page Reference	Summary and Comments
1	1975	7	Reasonable answers in P/S (Ch.4 pp.88-109)	p. 89 Very good introductory material on the idea of "about" including. i) basis of rounding ii) visual example of disc for estimation iii) exercises 'fixing' these concepts p. 93 Clear introduction to extending rounding to decimal fractions.
			Selecting a suitable unit of measure (pp.125, 126)	The need a) to choose a suitable unit and b) to round off to an acceptable level of accuracy is outlined. This is followed by some very good exercises which could form the basis for excellent discussion.
			Calculator	No listing in index.
2	1975	8	Estimation in Everyday Life	p. 63-65 Excellent illustrations and development of the use of estimation in everyday life, with good outline of use of the 'decomposition' method (see Siegel, 1981)
			Calculator	No listing in index.
3	1976	9	Squares and Square Roots	p. 141 Graphing is used as a method for finding approximation for square roots. Very clear, with good exercises. p. 144 Divide and average Method is explained in order to calculat successive approximations for a square root.
			Checking Reasonable Answers Using Rounding Off (In Skills Summary)	p. 384 A brief section explaining how to check decimal multiplication using rounding; clear explanations.

			Calculator	No listing in index
4	1979	10	Practicing Skill of Rounding Off	p. 31 Useful and clear review of rounding off i) to a number of decimal places ii) to a nearest "decimal unit" iii) to a number of significant digits
			Calculator	No listing in index

Table 15
Textual Material on Estimation, Approximation and Use of the Hand-Held Calculator in Selected Mathematics
Holt Mathematics 1, 2, 3 (Intermediate)

Book	Year of Publication	Grade	Topic and/or Page Reference	Summary and Comments
1	1976	7	Rounding (p. 22), 23	A chart is provided as a review of how to round numbers to the nearest one, ten, hundred, ... million. This is followed by a series of review exercises.
			Estimating (p. 23), 24	Rounding is then used as a procedure for estimating answers in each of the four operations.
			Rounding Off (pp. 99-100)	Measuring length is used to illustrate that measures are never exact and exercises are provided in rounding off measure and numbers to specified numbers of decimal places.
			Calculator	No listings in index.
2	1976	8	Rounding Whole Numbers and Decimals (p. 25)	Useful review of this procedure with summary exercises.
			Estimating (p. 30)	Outlines use of estimating as a way of placing the decimal point in multiplication.
			Decimal Quotients and Division of Decimals (pp.31, 32)	Illustrates division of decimals followed by rounding using several methods. Clear exposition.
			Calculator	No listing in index.
3	1978	9	Estimating and Rounding (p.4,5)	Reviews the rules for rounding, and illustrates use to check fo reasonable answers to computations and problems involving computation of whole numbers and decimals. Clear and well developed, with useful set of exercises.

			Rounding and Estimating (p.26)	Uses them for checking errors in computation.
4	1979	10	Rounding (p. 10, 11)	Reviews rounding off and introduces "even-odd" digit rule.
			Estimation (p. 13)	Review and practice in purposes and methods of estimating. e.g., planning a concert or holiday.
			Estimation and Significant Digits (p. 13)	Use of significant digits, and estimating reasonable answers.
			Irrational Expressions (p. 87, 88, 89)	Uses linear interpolation to obtain estimates for irrational expressions like 5 320
			Calculator	No listings in index.